Magnetohydrodynamic tokamak plasma edge stability

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Abstract

The edge of a Tokamak plasma is interesting due to its geometrical structure that is difficult to model either analytically or computationally, its tendency to form "transport barriers" with increased confinement of energy and momentum, and the edge-localised instabilities associated with transport barriers that threaten the lifetime of components in large Tokamaks. Ideal magnetohydrodynamics (MHD) is generally well understood, but only in the past decade has a good theoretical understanding emerged of MHD stability at the plasmas' separatrix when one or more X-points are present. By reviewing and discussing our theoretical understanding of ideal MHD stability of the plasma's edge, a clear picture emerges for its ideal stability. Conclusions are: ideal MHD will limit the width of strong transport barriers at the plasma's edge, a strong edge transport barrier will be associated with ELMs, ELMs will have a maximum toroidal mode number, will be preceded by smaller precursor instabilities, and can be triggered by sufficient changes to either the edge or the core plasma. Observations are made for the mechanisms responsible for edge transport barriers and ELMs, some leading to experimental predictions, others highlighting important open questions.

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I. INTRODUCTION

The purpose of this article is to summarise our theoretical ideal Magnetohydrodynamics (MHD) understanding for edge-plasma stability in tokamaks, and then to explore the predicted consequences for edge plasma behaviour. The review is restricted to ideal MHD because the MHD plasma model is arguably the best understood model for plasma behaviour, and is thought to be responsible for triggering the potentially damaging edge-localised instabilities (ELMs), that will need controlling in the large Tokamaks proposed for fusion power production. Ideal MHD is a good model to study because despite providing the simplest model for global plasma behaviour, much of the underlying physics incorporated in MHD will be important for more complex plasma models. For example, it is true in ideal MHD and more generally that both magnetic-field-line bending and plasma compression must remain small during an instability.

The paper consists of two main parts, Section II that reviews our understanding of edge-plasma stability, and Section III that discusses the expected consequences for plasma behaviour. Following a short discussion of the limitations of ideal MHD in Section IV, Section V summarises the conclusions.

II. BACKGROUND

There have been a number of independent studies that have explored the influence of an X-point in the separatrix on MHD stability. These have considered a variety of different instabilities, in various equilibria, with different modelling techniques that include: analytical studies, 1-d, 2-d, and 3-d numerical modelling. The results of these studies are summarised in Table I, from which there is seen to be overwhelming evidence for the stabilising influence of the separatrix on ideal MHD instabilities. The only possible exception (Ref. [1]), is considered at the end of this section. We will firstly discuss the stability of Mercier modes and Peeling modes in turn, before providing a simple physical explanation for the stabilising influence of the separatrix. Next Ballooning modes are considered, for which the picture is complicated by the tendency of Ballooning modes with a finite toroidal mode number to extend radially across many rational surfaces - so that the stability of the mode at the plasma's surface/separatrix, is affected by the stability of the mode deep within the plasma's edge. We finish with a discussion of the potentially conflicting article Ref. [1].

Mercier modes[2] are radially localised modes that can be driven unstable by strong pressure gradients, and whose stability must be tested on individual flux surfaces. For a given plasma equilibrium this requires the evaluation of an integral to calculate a parameter D_M that determines the stability of individual flux surfaces, for a given plasma equilibrium. Bishop[3] considered the stability of Mercier modes by numerically evaluating D_M for flux surfaces increasingly close to the separatrix of a model equilibrium with an X-point that he had previously developed[4], finding that they become stabilised near the separatrix. Mercier modes were also found to be stable near the separatrix of JT60. Webster[5] showed that if there is a non-zero current at the X-point, then $D_M \to 0$ near a separatrix, and hence that Mercier modes are always stable near the separatrix of any plasma with non-zero current at the X-point. The proof relied on a quantitative comparison of how fast terms such as the magnetic shear diverge near the separatrix, and for a non-zero current at the X-point it can be clearly shown that $D_M \to 0$ near the separatrix[5].

The stabilising influence of a separatrix on Peeling modes[6] was first noticed numerically[7, 8]. These results appeared to be in contradiction to analytical work by Laval et al[9] who considered an apparently arbitrary cross-sectional shape, and found that Peeling modes always had $\delta W < 0$, leading to the conclusion that they should always be unstable. To complicate matters, numerical calculations become increasingly difficult as the plasma boundary more closely approximates a separatrix, raising questions about the reliability of the stability calculations.

More recently Webster and Gimblett[10–12] showed analytically that the growth rate due to a Peeling mode instability asymptotes to zero as the plasma boundary more closely approximates a separatrix, with $\gamma^2 \propto q/q'$, where q is the tokamak's "safety factor"[13] and q'/q is proportional to the magnetic shear at the plasma boundary. This quantitative prediction has subsequently been confirmed by numerical calculations with ELITE[14, 15]. It was also found[10–12, 15] that whereas the growth rate asymptotes to zero, δW remained negative, in agreement with Laval et al[9]. An important point to remember for numerical calculations is the necessity of finding the most unstable mode numbers. This must be repeated as the plasma boundary is modified to more closely approximate the separatrix.

Within the high toroidal mode number (n) analysis of Refs. [10–12], for an equilibrium magnetic field $\vec{B}$ and a small plasma displacement $\vec{\xi}$, the stabilising affect of the separatrix and X-point is due to the inability to simultaneously keep $\vec{B}.\nabla\xi \lesssim 1$, $\nabla.\vec{\xi} \lesssim 1$, and $\left|\vec{\xi}\right|^2 \lesssim 1$, near the X-point. To keep $\vec{B}.\nabla\xi \sim 1$ we consider a mode $\xi \sim e^{im\theta - in\phi}$ with $(m - nq) \sim 1$ and $\theta = (1/q)\int^{\chi} \nu d\chi'$, where $\nu = IJ_\chi/R^2$ and J_χ is the Jacobian of the (ψ, χ, ϕ) coordinate system[2, 16]. However, within the high-n analysis to keep $\nabla.\vec{\xi} \sim 1$ also requires that $\xi_\perp \sim \partial\xi_\psi/\partial\psi$, where ξ_ψ is parallel to $\nabla\psi$ and $\xi_\perp$ is parallel to $\vec{B} \times \nabla\psi$, with ψ the usual poloidal magnetic flux function. The result is that $\xi_\perp \sim \partial\theta/\partial\psi \sim q'/q$, which diverges strongly at the separatrix with $|\xi|^2 \to \infty$. The divergence is sufficiently strong to require $\gamma^2 \to 0$ as we increasingly accurately approximate the separatrix. If we try to keep $|\xi|^2 \sim 1$, then either $\nabla.\vec{\xi}$ or $\vec{B}.\nabla\xi$ become large instead. The only way to avoid this strong stabilising influence from the X-point, appears to be by the mode becoming zero near the X-point. However the additional field-line bending required by such mode structures will generally exceed the relatively weak drive for instability that is responsible for Peeling modes, stabilising them. A strong enough pressure gradient could in principle overcome such extra field-line bending, although a dedicated study to provide a quantitative estimate of how large a pressure gradient and edge current are required, does not yet exist. Therefore it is not yet possible to say whether this is important. It is also possible that non-linear effects could enhance the pressure gradient sufficiently to allow an instability.

Because the stabilising influence of the X-point arises from the need to simultaneously keep $\vec{B}.\nabla\xi \lesssim 1$, $\nabla.\vec{\xi} \lesssim 1$, and $\left|\vec{\xi}\right|^2 \lesssim 1$, for high-$n$ modes at least, it appears to be a generic affect that does not apply solely to the Peeling mode. Therefore it would appear that the inclusion of one or more X-points in the plasma's separatrix will have a stabilising influence on all fluid model's of plasma instabilities, to a greater or lesser extent. Also notice that as the plasma boundary more closely approximates the separatrix, the growth rate asymptotes to zero, but never is zero. The appendix of Ref. [11] provides an alternative interpretation of how marginal stability is approached, finding that near a separatrix it is possible to keep $|\xi|^2 \ll 1$ provided the radial plasma displacement ξ_ψ and as a consequence δW, both tend to zero. Because this occurs with $\xi_\perp \neq 0$ and consequently $|\xi| \neq 0$ both finite and non-zero, the conclusion confirms that the Peeling mode is marginally stable and the calculation is arguably therefore inconclusive. It is uncertain whether the inclusion of non-linear terms will lead to a stable or unstable prediction for the Peeling mode, neither is it certain which non-ideal terms ought to be included at the separatrix, or what their affect will be. A conclusion of this review will be that whereas our understanding of linear ideal MHD stability near the plasma's edge is largely complete, our understanding of the plasma edge is not, and the question of how best to model the plasma-vacuum boundary remains open.

Finally we consider the influence of the separatrix and X-point on the stability of Ballooning modes[16]. The stability of Ballooning modes near a separatrix with an X-point was first considered by Bishop[3]. He considered infinite-n Ballooning mode stability, solving the 1-d differential equation to determine the Ballooning mode stability of individual flux surfaces in a model equilibrium he had previously developed[4]. The effect of the X-point was incorporated by considering flux surfaces that were increasingly close to the separatrix. Unfortunately, whereas he did appear to find a stabilising influence, he was unable to provide a conclusive result due to numerical difficulties near the separatrix. Ballooning mode stability near the separatrix was also considered for JT60 plasmas, and they were found to be stabilised[17].

An issue that was explored by Bishop[3] was how the poloidal location of the X-point affects Ballooning mode stability. Near the separatrix, but not so close that the mode was stabilised, Ballooning mode stability appeared to be influenced by the poloidal location of the X-point. The most unstable position was to have the X-point on the outboard side (the position of the X-point in JT60 prior to it being upgraded to JT60-U). A study of these affects for finite-n Ballooning modes was recently completed by Saarelma et al[15]. Interestingly, he found agreement with Bishop[3] that the poloidal location of the X-point would affect Ballooning mode stability, but unlike Mercier modes, Peeling modes, and infinite-n Ballooning modes, the X-point in the separatrix was not sufficient to stabilise the modes. The reason for this result arises from the radial extent of finite-n Ballooning modes and is discussed next.

It is known that finite-n Ballooning modes can extend into regions where the infinite-n mode is entirely stable[18] (see figure 1). Similarly, finite-n Ballooning modes can be non-zero at the separatrix, even if infinite-n Ballooning modes would be stable there. Whereas the X-point does have a stabilising influence on finite-n Ballooning modes, unlike Peeling modes, Mercier modes, and infinite-n Ballooning modes, this influence is insufficient to stabilise them completely. It is also found that the poloidal location of the X-point can modify the stability of finite-n Ballooning modes. This is particularly noticeable for modes near marginal stability, for which a mode's stability (or instability), can be changed by

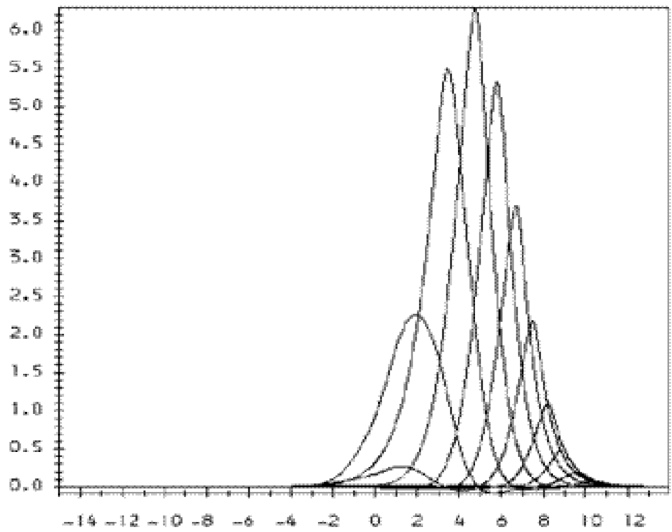

FIG. 1: A plot of a finite-n Ballooning mode versus radius, for an equilibrium with a minimum in q at $x = 0$, and negative/positive shear for x -ve/+ve respectively (see Ref. [18] for details). Regions with negative magnetic shear are stable to infinite-n Ballooning modes. As observed in Ref. [18], and shown above, finite-n Ballooning modes can be non-zero in regions where infinite-n Ballooning modes are stable. The same is true at the plasma's edge.

the poloidal location of the X-point[15]. However even with an X-point located in the most stabilising position, in general it is insufficient to ensure that Ballooning modes are stabilised[15]. The picture is illustrated in figure 2.

To summarise, ideal MHD predicts that:

1. The plasma is more stable (consequently with higher confinement), near the separatrix, whenever an X-point is present.

2. Instabilities require higher pressure gradients and a more radially extended structure to occur, leading to the expectation of a greater release of energy when they do.

These general predictions of ideal MHD match well with the tendency of plasmas to form transport barriers at the plasma edge, and for ELMs to intermittently eject large bursts of energy as opposed to transport rates simply increasing as the pressure gradient increases at the plasma's edge. These predictions are supported by a range of different ideal MHD modelling techniques, used by a variety of different authors as is summarised in table I.

The only paper that might disagree with the above picture is Ref.[1]. The authors studied the effect of an X-point at the separatrix by gradually increasing the "sharpness" of the

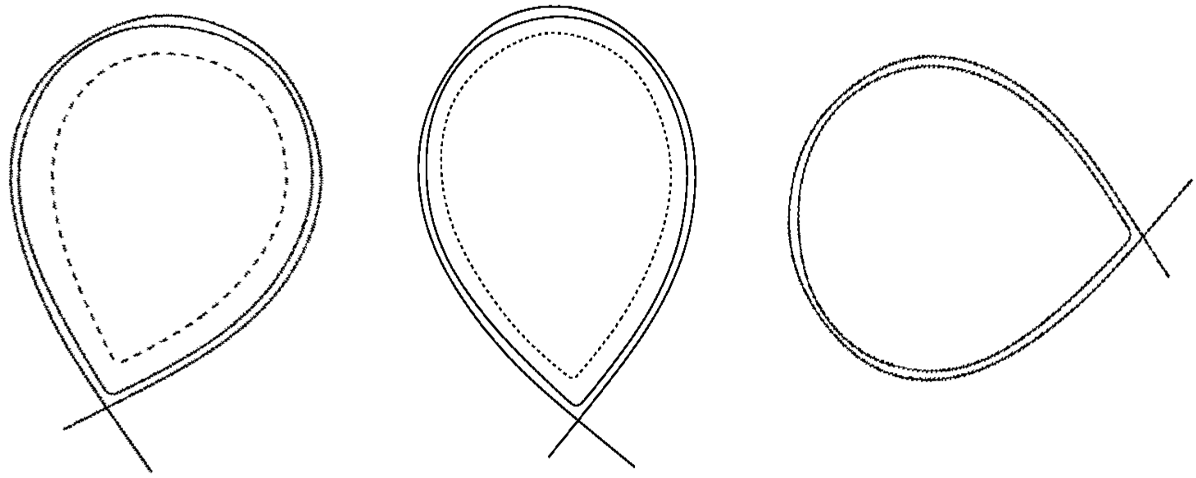

FIG. 2: The above cartoon illustrates the stabilising influence of the X-point. Sufficiently close to the separatrix the plasma is always stabilised. Near the edge but further from the separatrix, the radial extent of an equivalently stable region (denoted by dashed line), depends on the poloidal location of the X-point, with modes being less stable if the X-point is on the outboard ("bad curvature") side of the plasma.

plasma shaping so that it increasingly accurately approximated an X-point. Contrary to the results of the authors in Table I, they found that shaping the plasma to form an X-point caused the Ballooning and Peeling-Ballooning modes to be stabilised (see Summary in Ref. [1]), but did not affect the stability of Peeling modes. It is not clear why this is, some possibilities are suggested below. There are two aspects of this study that distinguish it from the studies in Table I: firstly that the authors determine stability by considering δW (as opposed to calculating the growth rate), and secondly that they consider the affect of shaping the plasma to form a *second* X-point in addition to one that is already present. Refs. [10–12] resolved the seeming discrepancy between the stable predictions of numerical calculations in Table I and the unstable prediction from an analytical calculation of Ref. [9] by noting that despite the Peeling mode's growth rate tending to zero as the boundary more accurately approximates a separatrix with an X-point, δW remains negative (which would usually indicate instability). Therefore we might expect the calculations of δW in Ref. [1] to find $\delta W < 0$ for Peeling modes, but this does not ensure that the Peeling mode is unstable because a calculation for the growth rate would be expected to find that despite $\delta W < 0$, the growth rate asymptotes to zero. Also, the way in which the extra X-point is added leads to stronger global (as well as local), shaping of the plasma. It is possible that this shaping is responsible for the stabilising influence on Ballooning and Peeling-Ballooning modes, and that any additional stabilisation from the formation of an *extra* X-point is a relatively small effect. Given these remarks, it is possible that the disagreements arise from studying the effect of an extra X-point on δW, as opposed to studying the effect on the growth rate of introducing an X-point into the equilibrium with a previously smooth boundary.

A. Recent developments

Ferraro et al[19] used an MHD code M3D-C^1 to study the influence of the separatrix on plasma stability. The code was firstly benchmarked against ELITE, then used to perform three studies in particular:

1. How plasma stability is affected by moving the jump in density at the plasma-vacuum interface, away from the separatrix.

2. Replacing the discontinuous jump in density at the plasma-vacuum interface with a non-uniform density profile that smoothly tends towards zero in the "vacuum".

3. Replacing the ideal MHD idealisation of zero resistivity in the plasma and infinite resistivity in the vacuum, with a non-uniform resistivity that smoothly increases as we move into the "vacuum".

Considering these in turn: (1) It was found that as the jump in density was moved further inwards away from the separatrix, modes became less stable. Moving the jump out past the separatrix had little effect. The destabilising influence of moving the density jump inwards from the separatrix is unsurprising if the picture presented in Refs. [10–12] and summarised here are correct. Ref. [10–12] found that the most unstable modes have a kinetic energy that is proportional to both the density and the magnetic shear, the latter of which diverges at the separatrix when an X-point is present. Therefore setting the density to zero before the magnetic shear diverges too strongly should reduce the kinetic energy term and increase the growth rate, as was found. (2) Replacing the jump in density at the separatrix with a non-uniform density was found to be slightly destabilising. Again this is to be expected because as noted in Ref. [19] a reduced density near the separatrix will reduce the kinetic energy term and increase the growth rate. (3) Replacing the jump in resistivity with a smoothly increasing resistivity was found to be stabilising. Because the jump in current at the plasma's edge provides a drive for instability, reducing the current gradient would also be expected to reduce the drive for instability, as was found.

The possibility that a strongly stabilised edge (assumed in this paper to be due to the separatrix's X-point), is responsible for both H-mode and ELMs, is consistent with recent gyrofluid simulations in Ref. [20]. Ref. [20] considered a circular cross-section plasma and found neither H-mode nor ELMs, instead as the plasma was made less stable the turbulent transport simply increased. In contrast, Sugiyama et al[21]'s MHD simulations of an equilibrium whose boundary was a separatrix with an X-point, claimed to see ELMs, although it is not clear why they regarded the instabilities to be "ELMs", as opposed to MHD turbulence. In fact they went further, suggesting that "Plasma edge instabilities provide a natural mechanism for creating and sustaining low levels of magnetic stochasticity in the plasma edge that could explain the steep edge pressure gradient of the H-mode and its requirement for a minimum level of plasma heating.". However this suggestion is not consistent with the ELM free H-modes found in the Li-divertor experiments discussed in Section III F, and therefore seems unlikely to generally to be the case. On the other hand, the possibility for a large ELM to provide the mechanism by which turbulence is temporarily suppressed for long enough to allow a transport barrier to form, could help explain why the transition from L-H mode is usually accompanied by a large ELM.

The suggestion of ELMs (albeit transiently), stabilising the plasma's edge is also present in the MHD simulations of Ref. [22], although the stabilisation was of an instability and not necessarily of transport. For this case the stabilisation was of an MHD Ballooning instability (thought to be the cause of the ELMs), and the same would not necessarily hold for small scale turbulent transport. It was found that an ELM would drive a strong poloidal flow, that appeared to prevent ELMs until the flow had decayed, after which another ELM and strong poloidal flow would be driven. Provided enough heating power were supplied to keep the system in an unstable state (for zero/small poloidal flow), then the cycle of ELM driving a stabilising poloidal flow that subsequently decays and allows an ELM, could continue. It would be interesting to know if the strong poloidal flow is driven by the divergence in $\xi_\perp$ needed to keep $\nabla.\vec{\xi} \sim 1$ and $\vec{B}.\nabla\xi \sim 1$ near the X-point. If this were the case then because $\xi_\perp$ would be expected to be much lower for equilibria without an X-point, then for those equilibria this ELM cycle would be modified. As with Ref [21], ELM free H-mode plasmas observed in Li-divertor experiments[23] demonstrate that a non-linear ELM cycle is not required for H-mode, supporting the suggestion that ELMs are a symptom of high confinement, not the cause of it.

III. DISCUSSION

Having considered the predictions of ideal MHD for plasma stability near a separatrix (with an X-point), we now explore the consequences for the plasma pedestal, edge stability, and ELMs. The discussion relies on a generic model whereby there is a strongly stabilised region near the plasma's edge, possibly caused by the combination of strong shaping associated with the X-point plus an edge transport barrier, and a strong pressure gradient in the transport barrier that provides the drive for instabilities. Much of the following discussion is summarised in figures 3, 4, and 5.

A. Max mode number for ELMs

The picture developed in the previous section requires the trigger for an edge instability to have a radially extended structure. More explicitly, if we require an ELM to have a non-zero displacement of the plasma surface, then an ELM requires a radially extended instability to overcome the strong stabilisation near the separatrix. This in turn requires a finite toroidal mode number n, that is sufficiently low to allow a sufficiently radially extended structure. For a given equilibrium profile, a stability analysis would determine the most unstable mode number n^* for which the edge plasma displacement is non-zero. Note that within the plasma's edge there can be more unstable modes with higher mode numbers, e.g. infinite-n Ballooning modes, but they will cause negligible (zero) plasma displacement of the plasma surface, and such instabilities would not usually therefore be considered to be ELMs. A non-zero edge displacement could be used as criterion to determine the maximum mode number needed to trigger an ELM.

B. Radial limit to transport barrier width

Conventional models for the plasma pedestal assume that the main role of instabilities is to limit and determine the pressure gradient in the pedestal. Whereas instabilities do place a maximum limit on the pressure gradient, what is observed and suggested here is that the most important role of instabilities is to place a limit on (and possibly determine), the width of the edge transport barrier. Near the separatrix high pressure gradients are stabilised by the strong shaping from the X-point, however if the high pressure gradient region extends sufficiently far radially into the plasma then an intermediate-low n instability can be driven unstable. Thus ideal MHD suggests the picture whereby Ballooning modes are driven unstable when the high pressure-gradient region associated with an edge transport barrier extends sufficiently far radially into the plasma to become unstable and trigger an ELM. In practice the stability of a mode is determined by the combination of pressure gradient, transport barrier width, and the edge current - changing any of these can affect stability. However if we assume that the edge currents do not change much, and that the pressure gradients are held fairly constant due to transport or other instabilities that limit their size, then we might expect the transport barrier width to be the dominant parameter. This is illustrated in fig. 3. Interestingly, the suggestion that an instability might limit the radial extent of a transport barrier was first made in the context of internal transport barriers (ITBs)[18]. In that context it was noticed that it was not possible to have a stable high pressure gradient region that extends all the way from the plasma's centre to the plasma's edge[18]. Here again we have a picture that limits the radial extent of a transport barrier, but where the stable high pressure gradient region is possible because of the stabilising influence of the X-point in the separatrix as opposed to the stabilising influence of negative magnetic shear that allows the formation of an ITB.

C. The plasma core affects edge stability

In the picture presented in the above Sections, the radially extended mode that is presumed to be responsible for ELMs is strongly stabilised near the plasma's separatrix (due to the strong shaping associated with the X-point), and is also stabilised by the lower pressure gradient region within the core plasma. The drive for instability is taken to come from high

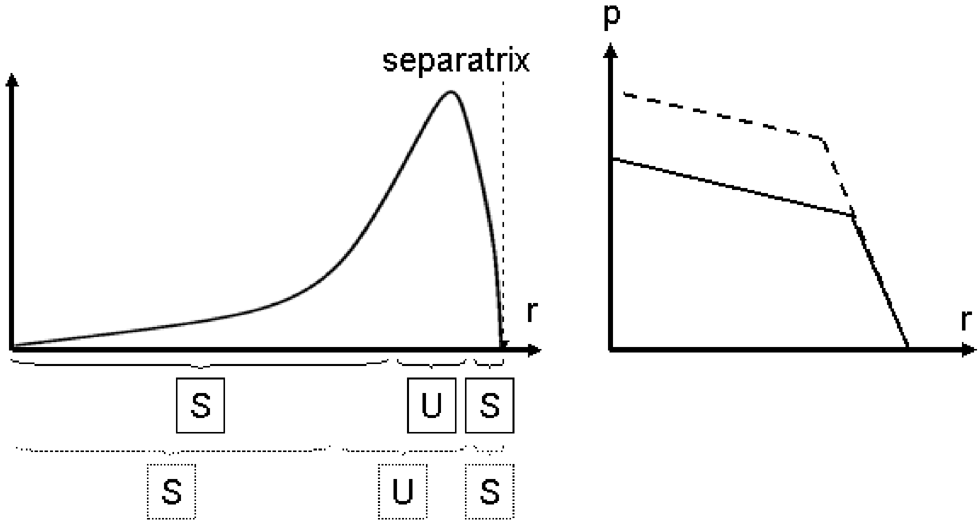

FIG. 3: The stability of radially extended modes that can cause a non-zero plasma displacement of the plasma surface (an ELM), is determined by the plasma stability of a radially extended region. If the radial extent of the transport barrier increases, then a high pressure gradient will exist in regions that are only weakly stabilised by the separatrix. If the radial extent of the transport barrier becomes sufficiently large, then the destabilising drive will exceed the stabilising influence from the X-point and the plasma core, driving an instability. In this way, instabilities can limit the radial extent of edge transport barriers.

pressure gradient region sufficiently far from the separatrix that an infinite-n Ballooning mode could become unstable at that radial location. If the core plasma becomes less stable, perhaps due to an increased pressure gradient, then not only does the radial extent of the unstable region increase slightly, but the stabilising influence from the core plasma is reduced. In practice an increase in core plasma pressure and the consequent changes to the equilibrium can both stabilise and destabilise the plasma's edge, but for this discussion we will assume that an increase in the core plasma pressure reduces its stabilising influence. If the stabilising influence from the core plasma is reduced sufficiently then the radially extended Ballooning mode can again become unstable. Either way, within this picture changes to the core plasma can affect the stability of the plasma's edge. This is illustrated in fig. 4. The different ways in which an ideal instability could trigger an ELM - either by destabilising the core plasma or by increasing the transport barrier's width, may be related to the different ELM properties and types. A prediction might be that it is easier to trigger ELMs

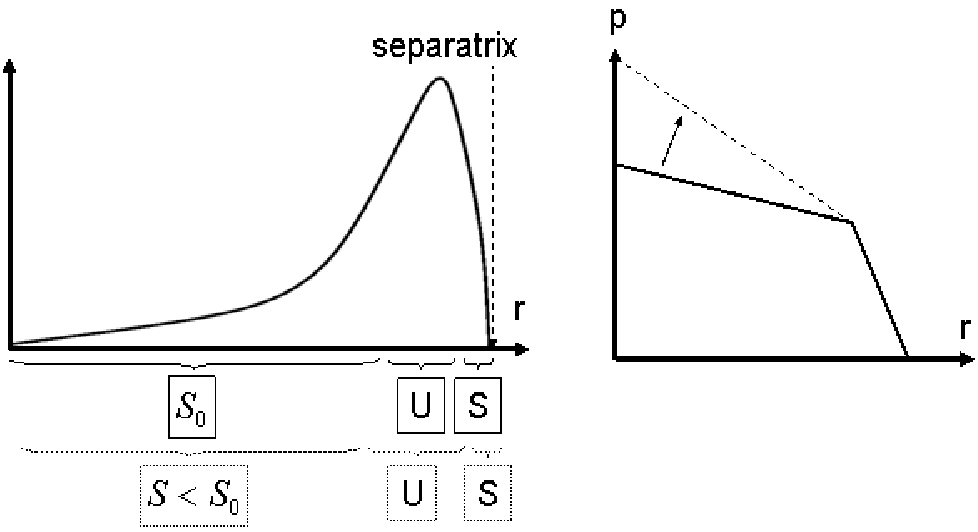

FIG. 4: If we assume a transport barrier of fixed radial extent (we do not consider what is limiting its radial extent), then a sufficient destabilisation of the core plasma can allow an instability to occur. This might for example be due to an increased pressure gradient, although changes to the core pressure gradient can in principle both increase and decrease the edge stability.

with a pellet that enhances the density gradient in the less stable region further from the separatrix, than with gas fueling, that would primarily increase the density gradient near the strongly stabilised separatrix.

D. H-mode, ELMs, and transport barriers

If the picture presented here is correct, it immediately explains why H-mode is associated with ELMs - either the transport barrier will grow in width until an instability is triggered, or the internal pressure gradient will increase until an instability is triggered - and the only modes with non-zero amplitude at the plasma surface will have a large radial extent. It also suggests that it might not be possible to prevent ELMs without accepting either a reduced pedestal width, or a reduced pedestal gradient. Because the radial region of enhanced stability is reduced if the X-point is moved further towards the outboard side of the plasma, then a prediction would be to expect both a weaker transport barrier and smaller ELMs if the X-point is located in a less stable region.

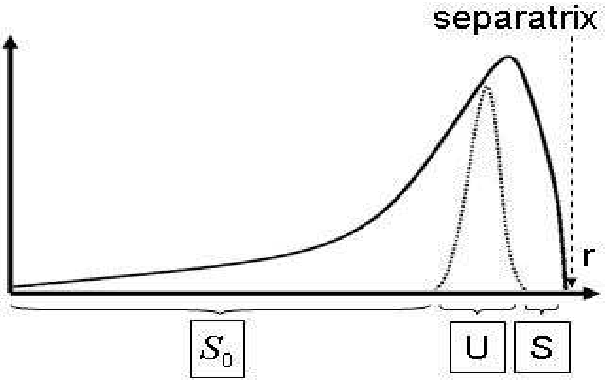

FIG. 5: In the picture where an unstable region grows sufficiently to drive an instability, a pre-requisite is for a radially localised region of unstable plasma. If such a region exists, then a sufficiently high mode number (radially localised) instability would be expected at that location. In other words, before an ELM is observed, the model predicts precursors, small radially localised instabilities with higher mode numbers than the final radially extended ELM.

E. Precursors to ELMs

An essential element in the picture presented here, is that in addition to a strong stabilising influence at the separatrix, there is a destabilising drive from the transport barrier's high pressure gradient. Sufficiently far from the separatrix, this high pressure gradient is presumed to be able to drive radially localised modes such as high-n Ballooning modes, to become unstable. Once this drive exceeds the stabilising influence from the lower pressure gradient core plasma and the stabilising separatrix at the plasma's edge, then the radially extended mode presumed to trigger an ELM can occur. Prior to an ELM however, as just noted, a region that is unstable to radially localised modes (e.g. infinite-n Ballooning modes), should form. Thus according to this model we would expect precursors, smaller plasma instabilities, prior to observing ELMs (this is illustrated in fig. 5). Because of their smaller radial extent, we would expect these precursor instabilities to have a mode number that is greater than or equal to that of the following ELM. It has been suggested[27] that because more modes can fit in a wider pedestal, that the possibility of consequently more precursors could lead to a reduced pressure gradient for wider pedestals.

F. Lithium divertor and limiter experiments

Recent experiments[23] considered the effect of a Lithium (Li) divertor surface on plasma performance[23]. Li is expected to improve the vacuum conditions and reduce recycling of particles from the vacuum back into the plasma, leading as a consequence to a reduced density gradient at the plasma's edge. For example, if the transport in the transport barrier is by diffusion or any other process for which the flux is proportional to the density gradient, then provided the diffusion proportionality constant remains roughly the same across the transport barrier's width, then conservation of particles requires the radial flux across the plasma surface to equal the radial flux within the transport barrier (to a first approximation). Therefore for such examples, the transport barrier's density gradient is determined by the density gradient at the plasma's surface - which is determined by recycling. This is illustrated more clearly in Appendix A. It is interesting that the reduction in density gradient in the transport barrier found in the Li-divertor experiments[23], is consistent with transport in the transport barrier being proportional to the density gradient. From the perspective of this article the main point is that the elimination of ELMs is consistent with the reduction in the transport barrier's pressure gradient - if there is an insufficient drive for instability then no ELMs will be observed. Whereas it is a not a conclusive indication that ELMs are a pressure-driven (as opposed to current driven) phenomena, and recent work[19] has highlighted the sensitivity of edge instabilities to the detailed edge-plasma properties, it is consistent with the hypothesis that high pressure gradients are required for ELMs. If a reduced pressure gradient is sufficient to prevent ELMs, then any mechanism that can sufficiently reduce the pressure gradient should therefore eliminate them.

The transport barrier in the Lithium experiments[23] did not extend across the entire plasma. It is not certain whether a weaker, more radially localised instability is limiting the transport barrier's radial extent by becoming unstable and increasing the transport, but this could be explored by testing the equilibrium's stability to infinite-n Ballooning modes for example, at the inner-most part of the transport barrier.

G. Limiter plasmas

A key experiment for the ideal MHD ELM model presented here, is the behaviour of plasmas in limiter H-mode plasmas[24] for which the plasma boundary does not have a conventional X-point. Firstly note that a limiter's influence on plasma behaviour is not yet fully understood. Also note that H-mode indicates the formation of a transport barrier, which usually indicates that the region has increased stability, although the exact mechanisms for transport barrier formation and sustainment are not yet confirmed. The model described in the previous sections works equally well regardless of the mechanisms causing the plasma's edge to be stabilised, so if a region of enhanced stability is forming at the plasma's edge then similar behaviour would be expected. Crucially, in the limiter H-mode experiments the observed ELMs were difficult to classify, suggesting the pedestal is different to the pedestal in plasmas with X-points in the separatrix. On the other hand, precursors to ELMs were observed[24], which is consistent with the picture of a strongly stabilised region near the plasma's edge and a transport barrier's strong pressure gradient driving instabilities.

H. The EPED1 pedestal model

The picture of ELMs presented here, based on the understanding of the ideal MHD stability of the plasma's edge, has similarities with the EPED1[25] model. This is unsurprising because the EPED1 model relies heavily on the ideal MHD stability of the plasma's edge, with some differences discussed below. A qualitative physical difference is that calculations with ELITE used in edge modelling such as EPED1, approximate the separatrix shape. The consequence is that Peeling modes remain unstable - this is not the case when the same calculations approximate the separatrix with sufficient accuracy. This is discussed further in Section III I below. A difference in interpretation is that EPED1 does not explicitly rely on a stabilised region near the plasma surface - although it does implicitly incorporate the effect. Another difference is that no thought is given in this paper to the pressure gradient in the pedestal - there will be maximum limits placed on it by instabilities and diffusive transport, but EPED1 explicitly suggests that the maximum pressure gradient for kinetic Ballooning mode instabilities determine the pressure gradient. Here it is pointed out that ideal MHD indicates that infinite-n Ballooning modes are stabilised near the separatrix, but might be observed near the inner edge of the transport barrier as precursors to ELMs. The stability of infinite-n Ballooning modes are often used to indicate how stable kinetic Ballooning modes are. It is also noted that the EPED1 model has limited applicability, for example it would incorrectly predict the pedestal properties in the Li divertor experiments discussed in Section III F, for which the observed pressure gradient is lower than in experiments without Li.

I. The Peeling-Ballooning Model

As discussed above, ideal MHD predicts that the plasma edge can be unstable to Peeling modes if the edge current is too high, or unstable to Ballooning modes if the edge pressure gradient is too large. The "Peeling-Ballooning" model provides a picture for describing the edge plasma's behaviour in terms of ideal MHD stability. Ideal MHD instabilities are taken to be the trigger for ELMs, and ideal MHD stability limits are taken to impose limitations on the maximum pressure gradient in edge transport barriers. The Peeling-Ballooning model suggests that the observed ELM size depends on whether it is triggered by a Peeling mode or a Ballooning mode, the latter of which has larger radial extent in the linear stability analysis. Thus within the Peeling-Ballooning model, a picture emerges whereby smaller ELMs are driven by Peeling modes, and larger ELMs are driven by Ballooning modes.

A problem with the picture that is clear from this article, is that within the ideal MHD model at least, the Peeling mode appears to be completely stabilised near the separatrix. In practice, stability calculations "cut-off" the plasma nearest to the separatrix, instead taking a plasma boundary that is a small distance within the separatrix, and hence continue to find unstable Peeling modes. Stability boundaries calculated with the ELITE code and others using this technique appear to be in reasonable agreement with the observed edge-plasma properties and ELM types observed in various experiments. This suggests that although the Peeling-Ballooning model is incomplete at present (within ideal MHD the Peeling mode should be completely stabilised at the separatrix), the good agreement with the observed plasma edge properties and ELM types in a variety of experiments (albeit with an implicit free parameter for where to "cut-off" the plasma boundary), provide sufficient reason to remain optimistic that the Peeling-Ballooning model can be developed into a first-principles predictive tool for plasma edge stability and the properties of ELMs.

The work of Ref. [19] suggests that with more realistic modelling of the plasma-vacuum boundary the stabilising influence of an X-point can become reduced, and that the plasma's edge stability can depend on the details of the plasma-vacuum interface. If this is the case it might help explain the intermittent nature of ELMs, but in the absence of a universal plasma-vacuum boundary (describing all separatrix plasmas in H-mode at least), then deterministic modelling of edge instabilities might not be practical. In these circumstances it should still be possible to: develop a statistical model for the plasma's edge, a qualitative understanding of the processes causing the instabilities, and provide a sufficient understanding of instabilities to allow the development of techniques to prevent them or reduce their size.

IV. LIMITATIONS OF MHD

Ideal MHD is a very simple model of plasma. It has the advantage of capturing many of the important features of plasmas, but is nonetheless a fairly simple approximation. The limitations of ideal MHD are highlighted by consideration of the thought experiment outlined in Ref.[5], where a dipole of two axisymmetric coils with equal but opposite currents are used to form an X-point with an otherwise arbitrarily small perturbation to the plasma equilibrium. It seems clear that an arbitrarily small perturbation cannot have an arbitrarily strong stabilising effect. The open question that remains is: What extra physics needs to be incorporated into ideal MHD to correctly model the plasma's edge? It might simply be that non-linear terms are required - the prediction of a marginally stable Peeling mode is an inconclusive prediction for its stability, or that the boundary should not be modelled as a sharp jump from plasma to vacuum, alternately are extra physical terms such as resistivity essential to correctly model the plasma's edge? These questions are important for future and on-going work.

V. SUMMARY

The primary purpose of this paper was to review the predictions of ideal MHD for the stability of the plasma's edge, partly to bring attention to the consistent evidence for the stabilising influence of a separatrix with an X-point, and partly to stimulate a greater awareness of our understanding of it. The overwhelming evidence for the stabilising influence of the separatrix and X-point on radially localised ideal MHD instabilities is presented in Table I. A simple physical explanation for the stabilising influence of the X-point is provided for high-n modes in terms of the need within ideal MHD to simultaneously keep $\vec{B}.\nabla\vec{\xi} \sim 1$, $\nabla.\vec{\xi} \sim 1$, and $\left|\vec{\xi}\right|^2 \sim 1$, for $\vec{\xi} \neq 0$ near an X-point. This generic stabilisation mechanism will always make the plasma more stable, even if it is insufficient to entirely suppress instabilities. More than one X-point is expected to make the effect more robust.

The ideal MHD model of edge stability leads to a picture for ELMs and the edge transport barrier that is consistent with a number of experimental observations, namely that: edge transport barriers form more easily in plasmas with a separatrix and X-point, edge transport barriers have a finite radial extent, transport barriers are associated with ELMs, an (equilibrium dependent) maximum mode number for ELMs, ELMs can be triggered by changes to the core plasma equilibrium, precursors to ELMs are to be expected. These qualitative predictions arise from a generic picture of a strong pressure gradient that can drive instabilities, and a strongly stabilised plasma edge - the strong shaping associated with an X-point is the mechanism predominantly discussed here, but the formation of an edge transport barrier will also help (such non-ideal effects have not been discussed). The strong stabilisation at the plasma's edge subsequently requires radially extended instabilities, that in turn require lower mode numbers, and are also likely to lead to larger instabilities. Strong currents can also drive instabilities, or at least reduce the plasma's stability. It is unlikely that ideal MHD is a sufficiently detailed plasma model to provide a full quantitative description for edge plasma behaviour. However as mentioned at the outset, the physics included by ideal MHD will strongly influence plasma behaviour, and consequently the model is likely to provide a qualitative guide at least for the expected plasma behaviour.

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Ideal MHD stability near a separatrix with X-point								
Authors & Publication Details	Bishop [3] (1986)	Akiba et al[17] (1988)	Webster [5] (2009)	Webster & Gimblett [10–12] (2009)	Medvedev et al[7, 26] [7, 26] (2003,2006)	Saarelma et al[15] (2011)	Husymans et al[8, 22] (2005,2007)	Ferraro et al[19] (2010)
Equilibrium:	Model	JT60	Arbitrary	Arbitrary	Model	Model	Model	Model/JT60
Technique:	1-d numerical	1-d numerical	Analytical proof	Analytical proof	2-d numerical	2-d numerical	2-d & 3-d numerical	3-d numerical
Mercier modes	S.	S.	S.$^{+}$	N.A.	N.A.	N.A.	N.A.	N.A.
Ballooning modes	uncertain (high-n)	S. (high-n)	N.A.	N.A.	see text	see text	see text	see text
Peeling modes	N.A.	N.A.	N.A.	S.*	S.**	S.*	S.**	S.$^{\#}$

TABLE I: The table summarises the results from stability calculations by independent authors. The first row indicates the lead author and paper, the 2nd row indicates the type of equilibrium studied, i.e. a special "model" equilibrium, the particular experimental (JT60) equilibrium, or an arbitrary equilibrium. The 3rd row indicates the technique used. The 4th, 5th, and 6th rows summarise the stability predictions near a separatrix from the different authors, and for the different instabilities of Mercier modes, Ballooning modes, and Peeling modes. The notation "N.A." or "Not Applicable", indicates that the mode was not studied by that author, "S" indicates a prediction that the mode is stable, "S^{+}" is a stable prediction with the caveat that a non-zero current is assumed at the X-point, "S*" indicates that the mode is stabilised, but only becomes marginally stable (this is discussed more fully in the main text). "S**" indicates a stable prediction, with the caveat that subsequent studies by Saarelma[15] with ELITE[14] have shown that with sufficient care to find the most unstable mode, an apparently stable prediction can subsequently be found to be marginally stable. "S$^{\#}$" indicates that although the separatrix was found to have a stabilising influence and the code was successfully benchmarked with ELITE[14], the extent to which modes were stabilised depended on the details of the plasma-vacuum interface. This is discussed more fully in the text.

APPENDIX A: THE IMPORTANCE OF EDGE PROPERTIES FOR PLASMA CONFINEMENT

To illustrate the importance of the plasma's edge on the edge transport barrier, we can consider diffusion of particles as a crude model for transport in a transport barrier. Taking a particle number density n, and a flux $\vec{F}$, then the change in number density is,

$$\frac{\partial n}{\partial t} = -\nabla.\vec{F} \tag{A1}$$

Therefore at steady-state with $0 = \partial n/\partial t$, and integrating Eq. A1 using Gauss' theorem, we get,

$$0 = \int_S \vec{F}.\vec{dS} \tag{A2}$$

For a closed surface surrounding an annulus of plasma this requires

$$\int_{S_1} \vec{F_1}.\vec{dS} = \int_{S_2} \vec{F_2}.\vec{dS} \tag{A3}$$

illustrating that at steady-state, conservation of number density requires that the flux out of any region equals the flux into it. For diffusion, $\vec{F_1} = -D_1 \nabla n$, with a diffusion coefficient D_1. Therefore taking S_1 as a surface in the transport barrier and S_2 as a surface at the plasma-vacuum interface, then we can see the strong dependence of the transport barrier's density gradient on the density gradient at the plasma's surface. If for the purpose of illustration we take the diffusion co-efficients to be constant, and the density gradients to be constant on a flux surface, then we explicitly find,

$$\left.\frac{\partial n}{\partial r_N}\right|_1 = \left.\frac{\partial n}{\partial r_N}\right|_2 \frac{\int_{S_2} dS_2}{\int_{S_1} dS_1} \tag{A4}$$

for which the density gradient is determined up to a geometrical factor by the density gradient at the plasma's surface. More generally Eq. A3 indicates that the rate of flux through a transport barrier equals the rate of flux across the plasma-vacuum interface. In a situation with enhanced recycling this net flux rate will be lower, and as a consequence so also will the flux through a transport barrier, leading to a higher density gradient in the transport barrier (for diffusive flux, although a transport barrier is arguably the only place in a Tokamak plasma where we might reasonably approximate transport to be by diffusion.). Alternately if the recycling is *reduced*, then the net flux leaving the plasma-vacuum surface would be *enhanced*, requiring an increased flux from the transport barrier and reducing its density gradient. Whether this is sufficient to explain the reduced density gradients in the Lithium experiments is an open question, but a reduced pressure gradient could be sufficient to explain the absence of ELMs if they essentially require a high pressure gradient - as is suggested in this paper.

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