A Sheath Model for Negative Ion Sources
Including the Formation of a Virtual Cathode

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Abstract. A one dimensional model of the sheath between the plasma and the wall in a negative ion source has been developed. The plasma consists of positive ions, electrons and negative ions. The model takes into account the emission of negative ions from the wall into the sheath and thus represents the conditions in a caesiated ion source with surface production of negative ions. At high current densities of the emitted negative ions, the sheath is unable to support the transport of all the negative ions to the plasma and a virtual cathode is formed. This model takes this into account and allows the calculation of the transported negative ions across the sheath with the virtual cathode. The model has been extended to allow the linkage between plasma conditions at the sheath edge and the plasma to be made. Comparisons are made between the results of the model and experimental measurements.

Keywords: negative ion source, plasma sheath, virtual cathode
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INTRODUCTION

Neutral beam heating and current drive will be necessary on future large fusion devices to meet the requirements of steady state plasma burn and plasma stability. These neutral beams will need to be of energies in excess of 1MeV and hence require negative ion precursors [1]. Volume production of negative ions cannot produce the required current densities and enhancement due to the addition of caesium in the source is required [2,3,4,5]. This enhancement is usually attributed to surface production of negative ions by atomic and ionic bombardment. These surface produced ions must make their way across the sheath and into the plasma before they can be extracted as a beam.

It has been shown through a variety of models that under the right circumstances this transport can be inhibited by the surface produced negative ions modifying the potential in the sheath to form a virtual cathode. The virtual cathode forms when the negative ion production at the wall is high enough that the associated space charge cannot be supported by the sheath and the space charge limit is exceeded. This is due to the production of negative ions at the wall being dominated by atoms rather than positive ions. This has been seen in PIC simulations by Wunderlich et al. [6], Hatayama [7] and Taccogna et al. [8]. Amemiya et al [9] produced an analytical model of electron injection into a plasma containing negative ions. This was simply adapted
to the case of negative ions emitted from the source wall by McAdams and Bacal [10] and allowed the calculation of the maximum current density before the formation of a virtual cathode. This adaptation gave good agreement with a PIC code simulation [6].

This model has been extended to take into account the formation of a virtual cathode [11] and also the finite energy of the negative ions created at the surface. A limitation of the model was that it used the densities of the plasma particles at the sheath edge, rather than those in the bulk plasma where any measurements would take place. The model has been developed further to include the effects of the pre-sheath region to link the particle densities in the plasma to those at the sheath edge. This allows a comparison between the model results and experimental data, providing a benchmark for the method.

**THE MODEL**

The problem is subdivided according to Fig. 1 into a virtual cathode region, a standard sheath region, a pre-sheath region and the plasma. The cathode, from where the negative ions are emitted represents the plasma grid. The model is detailed fully in [11] but the basic parts of the process are presented here. The plasma is not modeled itself in this system, but provides values for the different particle densities. Figure 1 also shows the different parameters involved in the model; \(j_b\) is the current density emitted by the surface, \(V_c\) is the cathode potential with respect to the potential at the sheath edge, \(V_k\) is the depth of the virtual cathode, \(V_m\) is the potential between the virtual cathode and the sheath edge, \(j_{b\text{max}}\) is the current density that is transported across the virtual cathode, \(U_0\) is the potential between the sheath edge and the plasma and the densities \(n_{0,p}, n_{e0,p}\) and \(n_{00,p}\) represent the ion, electron and negative ion densities at the sheath edge (0 subscript) and in the plasma (p subscript). At low production levels of negative ions at the wall there is no virtual cathode and the emitted current can be supported by the sheath and so \(V_c = V_m (V_k=0)\) and \(j_{b\text{max}} = j_b\). In an ion source, where the plasma grid is biased, the potential \(V_c\) is the difference between the bias potential and the plasma potential with respect to the reference of the bias voltage. The Poisson equations for the situations where the virtual cathode does not exist and when it does exist can be set up. Analysis of those equations [9,11] yields results for the transmitted current across the sheath. This analysis also allows the potentials themselves to be calculated but these are not reported here. Magnetic field effects due to the filter field are not considered in this 1d model.

The maximum transmitted current across the sheath in the presence of a virtual cathode is given by

\[
\begin{align*}
  j_{b\text{max}} &= \frac{2 n_{00} V_0 \left(1 + \frac{V_m}{V_0} \right)^{1/2} - 1}{\left(1 + \frac{V_m}{V_0} \right)^{1/2}} + n_{e0} T_e \left( \exp \left( \frac{-V_m}{T_e} \right) - 1 \right) + n_{n0} T_n \left( \exp \left( \frac{-V_m}{T_n} \right) - 1 \right) \\
  &\quad + \frac{2}{e} \left( \frac{M_b}{2e} \right)^{1/2} \left[ \left( V_m + U_b \right)^{1/2} - U_b^{1/2} \right]
\end{align*}
\]
where \( T_{i,e,n} \) are the electron and negative ion temperatures, \( V_0 \) is the initial energy of the positive ions crossing into the sheath \((V_0 = U_0 + T/2)\) and \( U_b \) is the initial energy of the emitted negative ions. This equation arises due to the condition that the electric field at the potential minimum in the virtual cathode is zero. If in this equation we set \( V_m = V_c \) i.e. no virtual cathode this represents the point at which the electric field at the cathode is zero i.e. the space charge limit and \( j_{\text{b max}} \) is the maximum transported current density before the virtual cathode is formed.

![Figure 1](image)

**FIGURE 1.** The virtual cathode, standard sheath, pre-sheath and plasma regions.

The densities and the maximum transported current density are related by the quasi-neutrality condition at the sheath edge

\[
n_e = n_0 - n_0 - \frac{j_{\text{b max}}}{e} \left( \frac{M_h}{2e} \right)^{1/2} (V_m + U_b)^{-1/2}
\]

and the initial energy of the positive ions is derived from the requirement of the derivative of the space charge density with respect to potential and given by,

\[
V_0 = \frac{n_0}{2 \left( \frac{n_e}{T_e} + \frac{n_n}{T_n} - \frac{j_{\text{b max}}}{e} \right)^{1/2} \left( 2V_m + 2U_b \right)^{-3/2}}
\]

When a virtual cathode exists the negative ions emitted from the cathode are retarded by the virtual cathode potential \( V_k \). It is assumed that the emitted current density, \( j_b \), and the current density at the minimum of the virtual cathode, \( j_{\text{b max}} \), are related by a simple exponential barrier i.e.
\[ V_k = T_b \ln \left( \frac{j_b}{j_{b\text{max}}} \right) \]  

(4)

where \( T_b \) is the emitted negative ion temperature, \( U_b = T_b/2 \).

The equations above involve the densities at the sheath edge but these can be related to those in the bulk plasma by equations such as

\[ n_{\text{i0}} = n_p \exp \left( - \frac{U_0}{T_e} \right) \]  

(5)

This linkage between the sheath edge and the bulk plasma was not present in the original model [11].

This set of equations can be solved iteratively. To reach a solution for the system the values of \( n_{\text{i0}} \), \( V_e \), the temperature of each species, \( j_b \) and the ratio between volume produced negative ion density and positive ion density, \( D \) are all required. The emitted current density, \( j_b \) can be calculated from the atomic hydrogen flux and the yield of negative ions for a particular atomic hydrogen temperature and the emitted beam temperature, \( T_b \), can be calculated from the kinetics of the conversion process [11]. The average energy \( E-E_{\text{thr}} \) for a Maxwellian for atoms with a temperature \( T_H \) is calculated where \( E_{\text{thr}} \) is the threshold energy for negative ion production. This threshold energy is equal to \( W-E_A \) where \( W \) is the work function, taken to be 1.5eV, and \( E_A \) is the electron affinity of a hydrogen atom (0.75eV). An energy reflection coefficient of 0.7 is also taken into account for any other energy loss in scattering from the surface. The value of \( D \) used throughout is 0.02, taken from [12]. The first part of the process is to guess a value of \( V_m \) which is then used to find the electron density at the sheath edge using the plasma neutrality condition. This is different to the process used in [11] where the value of \( n_{\text{i0}} \) was specified and \( n_{\text{i0}} \) was calculated. This method is preferable as the electrons are more mobile and will respond to the potential more than the other species. On the first iteration the value of \( n_{\text{i0}} \) is set to that of \( n_p \) but on subsequent iterations a calculated value is used, similarly on the first step the value of \( j_b \) is used for \( j_{b\text{max}} \) but on subsequent steps the calculated value is used. \( U_b \) is the initial energy of the emitted negative ions and expressed in terms of a temperature as \( T_b/2 \). Following this the initial energy of the positive ions is calculated according to Eq.3. Following this, the value of \( j_{b\text{max}} \) and subsequently a value for \( V_k \) from Eq.1 and Eq.4 are calculated. The potential difference between the plasma and the sheath edge is found from

\[ U_0 = V_0 - \frac{T_e}{2} \]  

(6)

and so the positive ion density at the sheath edge is found from Eq.5. This calculation of \( n_{\text{i0}} \) was not included in [11]. The other densities do not need to be recalculated separately from the sheath edge to plasma as they are already calculated from \( n_0 \) and obey quasi-neutrality, however when scaling from the sheath edge values to the plasma values for comparison with experiment they are corrected according to the same exponential as the positive ions to preserve quasi-neutrality.
All that remains of the convergence procedure is the calculation of the $n_{n0}$ from the value of $n_{i0}$ through Eq. 7

$$n_{n0} = Dn_{i0} \quad (7)$$

and the recalculation of $V_m$ through Eq. 8.

$$V_m = V_c + V_k \quad (8)$$

This value of $V_m$ is then mixed with the previous value and the process repeated until convergence is reached. The total negative ion density in the plasma can then be found by combining the volume negative ion density $n_{np}$ with the beam negative ion density at the sheath edge.

$$n_{b0} = \frac{j_{b_{max}}}{2e(V_m + U_b)} \quad (9)$$

The density from Eq. 9 is then scaled to the bulk plasma. Poisson’s equation can then solved for each region for the converged parameters to give the potential through the sheath (not reported here). The entire process is coded as an IDL program that runs on a standard UNIX workstation and solves in less than a second.

**APPLICATION OF THE UPDATED MODEL**

**Effects of the Inclusion of the Pre-Sheath**

The original sheath model used the densities at the sheath edge. These are not known whereas those in the bulk plasma can be measured. The scaling method outlined above allowed the bulk plasma parameters to be used with the sheath model thus allowing comparison with experiment. The densities at the sheath edge are lower than those in the plasma by the factor exp(-$U_b/T_e$). The capability of the sheath to support a given flux of negative ions is mostly determined by the positive ion flux from the plasma although leakage into the sheath of plasma electrons and negative ions will reduce this. Thus using a given density as the bulk plasma density should result in a reduction in the transported flux as the positive ion density at the sheath edge is reduced. This is illustrated in Figure 2 where the virtual cathode depth is calculated for the same positive ion density at the sheath edge (omitting one step in the iteration cycle) and in the bulk plasma. The equations above are solved using a positive ion density of $n_i = 3.5 \times 10^7$ m$^{-3}$, $T_e = 2$ eV, $T_i = 0.8$ eV, and an emitted current density of 500 Am$^{-2}$ and an emitted energy of 0.7 eV. No negative ions are assumed to exist in the plasma for the sake of simplicity. In the case where the conditions are applied at the sheath edge, as the cathode potential is made increasing positive with respect to the plasma, there is no virtual cathode formed until this potential difference is less than ~ 7V. When the conditions are applied in the bulk plasma the virtual
cathode is found to persist out to very high cathode voltages as expected due to the lower positive ion density.

![Graph](image.png)

**FIGURE 2.** Virtual cathode depth as the cathode potential is varied for 500 Am⁻² emission at the surface for the case where the plasma conditions are imposed at the sheath edge (solid line) and in the bulk plasma (dashed line)

![Graph](image.png)

**FIGURE 3.** Relationship between the positive ion density in the plasma and the maximum current density before a virtual cathode can form for the plasma conditions given in the text.

To demonstrate the effect of the positive ion density on the virtual cathode directly the code was used for a range of \( n_p \) and a range of \( j_b \) for a \( V_c \) of -10V, generally if the virtual cathode has not been removed for this level of bias then it will persist indefinitely. The maximum level of emitted current density \( j_b \) that could be transported without a virtual cathode being present was calculated and the results are shown in
Fig. 3 for the plasma conditions used above. As the positive ion density increases then the positive ion flux to the wall increases. This allows more negative ions to flow from the wall since each positive ion compensates for the presence of a negative. This is not the complete picture since electrons and negative ions from the plasma add to the overall negative space charge in the sheath.

Comparison with Experiment

In order to test the model against experiment, as complete a data set as possible for the IPP BATMAN source has been assembled from the literature [13,14,15]. These data cover source pressures of 0.3, 0.4 and 0.5 Pa and RF powers of 38-78kW and consists of electron, positive and negative ion densities, electron temperatures, atomic hydrogen density. The input data is shown in Figure 4.

![Input data from IPP BATMAN](image)

**FIGURE 4.** Input data for the model from IPP BATMAN: (a) positive ion density (measured), (b) electron temperature (measured) and emitted negative ion current density (calculated from data).

It was assumed that the atomic hydrogen temperature was equal to the electron temperature and that the positive and negative ions both had a fixed temperature of 0.8eV. The yield of negative ions for a given atomic temperature was taken from the work by Seidl et al [16]. For an atomic temperature of 0.8eV, which is used in
previous simulations [6] and is typical of this ion source at the reported conditions, the yield is 0.12. From this data the emitted current density of negative ions from the surface could be calculated [11]; a value of $U_b$ of 0.7eV was used. The method of comparison was to take as input to the model, the measured positive ion density, measured electron temperature and calculated emission current density. Then the electron and negative ion densities were calculated for comparison with the experimentally measured values. Also calculated were the transported current density of negative ions across the sheath and the depth of the virtual cathode; these parameters are not presently measurable. From the data it was not clear what bias voltage was used and a value of $V_c = -1$V was assumed based on normal operating point of just below plasma potential. The comparison between the model and the experimentally measured electron and negative ion densities are shown in Figure 5. In all cases the calculated plasma neutrality was better than 0.05%. The model is in excellent agreement with the measured electron density. Agreement with the measured negative ion density is very good at the lower pressures but overestimates this density at the highest pressure. The model does not take into account any volume production of negative ions other than to fix their density as a fraction of the positive ion density. Furthermore no collision processes have been accounted for in this version; these may be important at the higher pressures. Given the uncertainty in some values such as bias voltage, and the ion and atomic temperatures the agreement with the model is very good.

![Figure 5](image_url)

**FIGURE 5.** Comparison of model with measured parameters from the IPP BATMAN source: (a) electron density and (b) negative ion density. Measured values are the symbols, calculated are the lines: solid is 0.3Pa, dashed is 0.4Pa and dotted is 0.5Pa

In Figure 6 the calculated transported current density across the sheath and the depth of the virtual cathode are plotted. The virtual cathode depth is highest for the lowest operating pressure. This is due to the lower positive ion density since the emitted current density is lowest for this pressure.
FIGURE 6. Calculated virtual cathode depth and the transported negative ion current density.

FIGURE 7. The dependence of virtual cathode depth against the cathode potential for different values of positive ion temperature for 0.3Pa and 69kW of RF power.

Positive Ion Temperature Effects

The positive ion and negative ion temperatures are not known. Thus a sensitivity study was carried out into the effect of varying the positive ion temperature since the positive ion flux is important in determining the virtual cathode depth and the transported flux of negative ions. This was done by varying the cathode voltage for different values of $T_i$ for the case of 0.3Pa source pressure and 69kW of RF power. The results of this can be seen in Fig. 7. In these cases a variation in virtual cathode depth of around 0.5V can be seen with $T_i$. The positive ion flux that is transported across the sheath depends on the initial positive ion energy as given by Eq. 6; the
lower the positive ion temperature the lower the flux in the sheath. At higher positive ion temperatures a higher negative ion flux can be transported across the sheath and thus the virtual cathode depth is lower.

CONCLUSIONS

A model of virtual cathode formation in the presence of surface negative ions has been further developed and then tested against experimental results. The model is able to predict experimental measurement very well given the uncertainties in some of the assumptions which have to be made. It has been shown that given a high enough surface emission of negative ions a virtual cathode will persist at very large cathode voltages. Further to this, the positive ion density has an essentially linear effect on potential negative ion flux that can be sustained from a surface without losses to a virtual cathode. The effects of positive ion temperature on virtual cathode formation have also been shown. The study has emphasized the need for comprehensive diagnostics of the ion source plasma.

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