Assessing the Merits of Resonant Magnetic Perturbations with Different toroidal Mode Numbers for Controlling Edge Localised Modes
Enquiries about copyright and reproduction should in the first instance be addressed to the Culham Publications Officer, Culham Centre for Fusion Energy (CCFE), Library, Culham Science Centre, Abingdon, Oxfordshire, OX14 3DB, UK. The United Kingdom Atomic Energy Authority is the copyright holder.
Assessing the Merits of Resonant Magnetic Perturbations with Different toroidal Mode Numbers for Controlling Edge Localised Modes

I.T. Chapman¹, A. Kirk¹, R.J. Akers¹, C.J. Ham¹, J.R. Harrison¹, J. Hawke², Y.Q. Liu¹, K.G. McClements¹, S. Pamela¹, S. Saarelma¹, R. Scannell¹, A.J. Thornton¹
and the MAST Team

¹EURATOM/CCFE Association, Culham Science Centre, OX14 3DB Abingdon, UK.
²FOM Institute DIFFER Dutch Institute for Fundamental Energy Research, 3430 BE Nieuwegein, Netherlands

Assessing the merits of resonant magnetic perturbations with different toroidal mode numbers for controlling edge localised modes


This is an author-created, un-copyedited version of an article accepted for publication in Nuclear Fusion. IOP Publishing Ltd is not responsible for any errors or omissions in this version of the manuscript or any version derived from it.

The Version of Record is available online at 10.1088/0029-5515/54/12/123003
Assessing the merits of Resonant Magnetic Perturbations with different toroidal mode numbers for controlling Edge Localised Modes

IT Chapman\textsuperscript{1}, A Kirk\textsuperscript{1}, RJ Akers\textsuperscript{1}, CJ Ham\textsuperscript{1}, JR Harrison\textsuperscript{1}, J Hawke\textsuperscript{2}, YQ Liu\textsuperscript{1}, KG McClements\textsuperscript{1}, S Pamela\textsuperscript{1}, S Saarelma\textsuperscript{1}, R Scannell\textsuperscript{1}, AJ Thornton\textsuperscript{1} and the MAST Team

\textsuperscript{1}CCFE, Culham Science Centre, Abingdon, Oxon, OX14 3DB, UK
\textsuperscript{2}FOM Institute DIFFER. Dutch Institute for Fundamental Energy Research, 3430 BE Nieuwegein, Netherlands

Abstract. ELM mitigation has been demonstrated in MAST using resonant magnetic perturbations (RMPs) with toroidal mode number, \( n_{\text{RMP}} = 2, 3, 4, 6 \). It has been observed that the mitigated ELM frequency increases with the amplitude of the applied field provided it is above a critical threshold. This threshold value depends on the mode number of the RMP, with higher \( n_{\text{RMP}} \) having a larger critical value. For the same ELM frequency, the reduction in the peak heat load on the divertor plates is approximately the same for all RMP configurations. The RMPs give rise to perturbations to the plasma shape, with lobe structures occurring due to the tangled magnetic fields near the X-point, and corrugations of the plasma boundary at the midplane. The X-point lobe length increases linearly with the applied field when above a threshold, with RMPs of higher toroidal mode number giving rise to longer lobes for the same applied resonant field. Similarly, the midplane displacements increase with the applied field strength, though the corrugation amplitude is less dependent upon the RMP configuration. For all \( n_{\text{RMP}} \), the RMPs result in enhanced particle transport and a reduction in the pedestal pressure gradient caused by an increased pedestal width, which is found to be consistent with a decrease in the critical pressure at which infinite-\( n \) ballooning modes are driven unstable in non-axisymmetric plasmas. The plasma rotation braking is strongest for lowest \( n_{\text{RMP}} \) whilst both the fast ion loss enhancement and the degradation of access to H-mode resultant from the application of RMPs are non-monotonic in \( n_{\text{RMP}} \), with the optimal case for both occurring for \( n_{\text{RMP}} = 4 \). Whilst there are advantages and disadvantages for all RMP configurations, the configurations found to be optimised in terms of pedestal degradation, access to H-mode, fast ion losses, plasma rotation and distortion to the plasma configuration in MAST are \( n_{\text{RMP}} = 3 \) or 4, consistent with the configurations anticipated for use in ITER.
1. Introduction and Background

Edge localised modes (ELMs) occur at the periphery of tokamak plasmas which operate in a high-confinement regime [1]. In this regime of improved confinement, a transport barrier, or ‘pedestal’, forms at the edge, leading to a strong pressure gradient. The high gradient in the pressure and current density which occur in the pedestal region drive peeling-ballooning instabilities [2, 3], which are believed to be manifest as ELMs [1]. However, empirical scaling of the energy released by ELMs suggests that the heat loads on the plasma facing components in ITER will be intolerable unless mitigated. In order to ensure an appropriate lifetime of plasma facing components in ITER, robust ELM control is required, either suppressing the ELMs completely or, at least, reducing the heat flux per ELM which is incident on the divertor plates [4]. One such control scheme is the application of resonant magnetic perturbations (RMPs), which perturb the magnetic field in the pedestal region, locally degrading the confinement.

RMPs have been applied to completely suppress ELMs in DIII-D [5, 6] and KSTAR [7], or to mitigate ELMs – that is to say increase their frequency and reduce their amplitude – in ASDEX Upgrade [8, 9], MAST [10, 11, 12] and JET [13, 14]. It is expected that ITER will use RMPs to control ELMs, with the non-axisymmetric fields having a toroidal periodicity of $n = 3$ or $n = 4$ [15]. This choice of mode number was based on DIII-D results which showed that $n = 3$ fields could suppress ELMs [5, 19] and calculations which suggested that in-vessel coils could provide sufficient perturbation to the plasma edge magnetic topology to have the same affect in ITER [17, 18, 16]. The in-vessel control coils for ITER were specified upon the basis of vacuum modelling [17, 18] which stipulated the necessary fields such that the RMP-induced islands overlapped across a certain width of the minor radius which was found to be correlated with ELM suppression in DIII-D experiments [19]. The present ITER design [20] has a set of 27 in-vessel six-turn coils (above, below and at the midplane) permitting a maximum current carrying capability of 90 kA-turns with a variety of field spectra possible since each coil has its own power supply. Configurations giving $n < 3$ RMPs were originally deprecated since the field penetrates further as the toroidal mode number decreases, giving rise to concerns about the interaction with intrinsic error fields, inadvertent deleterious effects on the core plasma or even the seeding of locked modes and ensuing disruptions. However, recently low-$n$ RMPs have been used to suppress ELMs in DIII-D ($n = 2$) [21] and KSTAR ($n = 1$) [7] without significant degradation of the core performance, opening the question of what the optimal configuration is for ITER.

With its 18 in-vessel coils [22], MAST is able to apply RMPs with toroidal mode numbers $n = 1, 2, 3, 4, 6$, making it ideally suited to investigate the issue of the optimal toroidal mode number for controlling ELMs. In this paper, we document the effect of RMPs in configurations with a range of toroidal mode numbers when applied in MAST and compare with numerical simulations of 3D equilibria and resistive MHD stability. In section 2 the effect of different RMP configurations on the ELM behaviour in MAST is documented, together with an assessment of the effect on the mitigated divertor heat.
flux. The changes in the plasma configuration, both in terms of lobes near the X-point and the midplane corrugations, due to RMPs with different configurations is outlined in section 3. Section 4 discusses the change in the pedestal structure associated with the RMPs and analyses the stability of the plasma edge for different \( n_{RMP} \). Finally, the toroidal rotation braking, degradation of access to H-mode and enhancement of fast ion loss resultant from RMPs with different periodicities is outlined in section 5. The implications of these various effects for the choice of RMP configuration in ITER is discussed in section 6.

2. Effect of different RMP configurations on ELMs and heat loads

Complete ELM suppression has not been achieved in MAST plasmas, but ELM mitigation – that is to say a marked increase in ELM frequency and commensurate reduction in the divertor heat load per ELM, coming at the price of a drop in plasma confinement – has been achieved with RMPs with a range of toroidal mode numbers in both lower single null diverted (SND) or connected double null diverted (DND) magnetic geometry. Figure 1 shows an example of the effect of the application of RMPs with toroidal mode number \( n_{RMP} = 2, 3, 4, 6 \) on an SND H-mode plasma with a plasma current \( I_p = 400\text{kA} \), an on-axis toroidal magnetic field \( B_{T0} = 0.55\text{T} \) and an edge safety factor \( q_{95} = 3.8 \) when heated with 3.6MW of NBI power. It should be noted that the \( n = 1 \) locked mode threshold is such that there is no window whereby ELM mitigation is achieved and disruptions are avoided in MAST. The change in ELM behaviour can be seen from the changes in the \( D_\alpha \) emission when the current in the in-vessel coils is applied for different RMP configurations. Also shown is the enhanced particle transport – or so-called density pump-out – that occurs when RMPs are applied as well as the toroidal rotation velocity towards the top of the pedestal. The applied fields with different \( n_{RMP} \) reach different magnitudes since for lower-\( n_{RMP} \) a back transition to L-mode is triggered at smaller \( I_{coil} \) (and correspondingly smaller resonant radial field \( b_{res} \)), as discussed further in section 5.

The applied resonant field for these different configurations is illustrated in figure 2 for both the same in-vessel coil current and when the coil current is taken from the MAST cases shown in figure 1. The field for these different RMP configurations is calculated using the ERGOS vacuum field line tracing code [23]. The effective radial resonant field component of the applied perturbation normalised to the toroidal field \( (b'_{res}) \) is the amplitude of the resonant Fourier component in the spectrum of \( b^1 \) divided by the average value of \( \nabla \sqrt{\psi_{pol}} \) on the magnetic surface, normalised to the major radius [23]:

\[
b'_{res} = 2 \frac{|b^1_{m,n}|}{R_0 \langle \nabla \sqrt{\psi_{pol}} \rangle}
\]

Near the plasma edge, the \( n_{RMP} = 4, 6 \) RMPs give the largest resonant field, with \( n_{RMP} = 2 \) giving the weakest resonant field, which should be expected given the current
in the in-vessel coils was much lower for the \( n_{RMP} = 2 \) case. Indeed, the \( n_{RMP} = 2 \) field from the RMPs is of similar magnitude to the intrinsic \( n = 2 \) error field measured in MAST.

It is possible to vary the effective resonant field at the plasma edge in MAST by varying the current in the in-vessel coils and also by moving the plasma outboard boundary closer towards the coils. Figure 3 shows the ELM frequency normalised to the unmitigated ELM frequency as a function of the radial field calculated with vacuum field line tracing at \( \psi_N = 0.98 \) for MAST plasmas with \( n_{RMP} = 3, 4, 6 \) RMPs applied. Note that the window between the applied field being sufficiently large to affect the ELM behaviour and the field required to induce a back-transition for \( n_{RMP} = 2 \) is very small, so we do not include \( n_{RMP} = 2 \) in this dataset. It is evident that the ELM frequency increases as a linear function of the applied field provided it is above a critical threshold, which itself is different for the different toroidal mode numbers. The critical \( b^r_{res} \) required for ELM mitigation scales with \( n_{RMP} \), with \( n_{RMP} = 3 \) being lowest.

It is observed in MAST that the energy released per ELM, the fractional density loss per ELM and the peak heat flux on the divertor plates decrease as the ELM frequency increases [11]. However, whilst the peak heat flux does scale linearly with the increasing ELM frequency, there is an offset meaning that as \( \Delta W \to 0 \), the peak heat flux remains above the inter-ELM value [30]. Here we consider the relative merits of the various RMP configurations for mitigating the heat flux on the targets. Figure 4 demonstrates that the peak heat flux to the outer strike point as measured by infrared (IR) thermography...
scales with the total energy incident on the divertor for resonant magnetic perturbations with \( n_{RMP} = 3, 4, 6 \). This intransigence with RMP periodicity is consistent with the heat flux mitigation resulting from the RMPs reducing the drive for the peeling-ballooning modes, but not changing the nature of the perturbation, that is to say the mitigated ELMs retain their type-I ELM characteristics [11, 12]. The different RMP configurations do lead to different strike point patterns on the targets, as discussed later in section 3, but do not affect the peak heat flux resultant from the mitigated ELMs. The reduction in peak heat flux is observed to scales with the \( \Delta W_{DIV} \) for all cases of \( n_{RMP} \). Whilst lower peak heat flux is shown in figure 4 for \( n_{RMP} = 4, 6 \) than \( n_{RMP} = 3 \), this is simply because the ELM frequency is lower in these shots for the level of applied field, since the \( b'_{res} \) is lower (see figure 2). Since the avoidance of damage to in-vessel plasma facing components is the primary motivation for the application of RMPs, if one configuration leads to optimal peak heat flux mitigation, this would almost certainly result in that configuration being used. However, since that is not the case, the choice of optimal RMP configuration in ITER will relate to the second order effects discussed in the following sections.
Figure 3. The ELM frequency as a function of the resonant radial magnetic field applied from the in-vessel coils at the nearest rational surface to $\psi_N = 0.98$ calculated with a vacuum approximation. The ELM frequency increases linearly with the applied field above a threshold for all toroidal mode numbers. The threshold in applied field increases with $n_{RMP}$.

Figure 4. The peak heat flux to the outer strike point measured by IR thermography as a function of the total energy to the divertor for different RMP configurations.
3. Effect of different RMP configurations on plasma configurations

3.1. Lobes near the X-point

When non-axisymmetric fields are applied, the resultant tangled magnetic fields are predicted to lead to the formation of lobe structures near the X-point [24]. Such structures have been directly observed in MAST [10, 25] and DIII-D [26], and used to explain the splitting of the divertor leg footprints observed on strike-point targets in various experiments with RMPs (see for instance references [27, 28, 29]). The lobe structures have very different characteristics depending on the configuration of the RMP applied. Figure 5 shows images of the plasma in the vicinity of the lower X-point for \( n_{RMP} = 3, 4, 6 \) in MAST single-null plasmas (as shown in figure 1) as measured by the filtered visible-imaging camera. The 3D location in the vessel where the camera viewing chords are tangent to toroidal surfaces are calculated using a ray tracing code. It is assumed that the majority of the light detected at a given pixel originated in close proximity to this tangency position, in order to localise the measurement. Unperturbed equilibrium flux surfaces are mapped to each pixels tangency location, allowing the image to be mapped in terms of normalised flux. For all \( n_{RMP} \), the radial extent of the lobes increases linearly with the resonant field applied [25]. The lobes resultant from the application of higher-\( n_{RMP} \) RMPs have narrower poloidal extent, but increased radial extent.

Figure 6 shows the images from 5 mapped to the equilibrium toroidal flux for RMPs with the same \( b_{res}' \). The apparent size of a lobe is measured by examining the variation of the \( He^{1+} \) light emitted from the plasma as a function of the distance from the separatrix, at a given poloidal angle [25]. The camera data is interpolated onto a regular mesh of poloidal angle and radial distance from the last closed flux surface for analysis purposes. It is clear that the size of the lobes – defined to be the distance over which the light intensity falls to 1/3 of its peak (separatrix) value – varies with \( n_{RMP} \), with the lobes being longest when an \( n_{RMP} = 6 \) field is applied.
Merits of RMPs with different toroidal mode numbers

Figure 6. The camera images shown in figure 5 mapped as a function of poloidal angle and equilibrium flux. The black dotted lines illustrate where the light detected by the camera fell to half of the peak value near the separatrix, before the RMP was applied but the plasma was still in H-mode. Note that the $n_{RMP} = 6$ case has slightly more input power, meaning that the edge is slightly hotter so the region of the SOL which emits $C^{2+}$ light will be slightly extended, but nonetheless, the longer lobes are consistent with field line tracing [25].

3.2. Corrugation at the midplane

As well as giving rise to lobe structures near the X-point, the application of RMPs in MAST also gives rise to a toroidal corrugation of the plasma midplane [22, 31, 32]. These displacements have been measured in MAST double null configuration with $n_{RMP} = 3$ and $n_{RMP} = 4$ RMPs applied (with $n_{RMP} = 6$ leading to negligible displacement in a double null configuration since the plasma is too far from the lower set of coils used to create an $n_{RMP} = 6$ field). We use double-null plasmas in this section of the paper (compared to the single-null plasmas elsewhere) since MAST has a number of diagnostics capable of measuring the plasma boundary with sub-cm radial resolution at the midplane located in various toroidal sectors around the vessel. This allows a toroidal map of the boundary position to be measured for different phases of applied field [22]. Figures 7 and 8 show the midplane boundary position as a function of the toroidal angle when $n_{RMP} = 3$ and $n_{RMP} = 4$ RMPs are applied respectively. The boundary corrugations are measured by the Thomson scattering diagnostic, visible imaging cameras, a linear $D_\alpha$ camera, the charge exchange recombination spectroscopy diagnostic, and an RGB camera. These figures show that upon application of the RMPs, the plasma boundary experiences a toroidal corrugation with periodicity and phase dictated by the applied non-axisymmetric field. For both the $n_{RMP} = 3$ and $n_{RMP} = 4$ RMPs, the displacement of the boundary is of the order of $\pm 2\text{cm}$, equivalent to $\pm 4\%$ of the minor radius [31].

The displacements have been shown to be in good accordance with those predicted by a 3D equilibrium reconstruction using the VMEC code [22]. Accordingly, we have used VMEC to predict the midplane corrugation amplitude for a range of $n_{RMP}$ and $b_{res}^r$. Figure 9 shows the position of the plasma boundary as a function of toroidal angle predicted by VMEC when $n_{RMP} = 2, 3, 4, 6$ RMPs are applied in a MAST double-null plasma for the case with full current in the in-vessel coils, and for a case with matched $b_{res}^r$. 
Figure 7. The RMP-induced midplane displacement as a function of toroidal angle as measured by different diagnostics in MAST when two phases of an $n_{RMP} = 3$ RMP are applied compared to the case without an RMP applied. The absence of an applied field shows the axisymmetric approximation is valid, whilst the RMPs lead to a demonstrable corrugation of the midplane plasma boundary of $\pm 2.5 \text{cm}$.

Figure 8. The RMP-induced midplane displacement as a function of toroidal angle as measured by different diagnostics in MAST when two phases of an $n_{RMP} = 4$ RMP are applied compared to the case without an RMP applied. The absence of an applied field shows the axisymmetric approximation is valid, whilst the RMPs lead to a demonstrable corrugation of the midplane plasma boundary of $\pm 2 \text{cm}$.

at the plasma boundary. It is evident that for the same applied current in the in-vessel coils, the lower-$n_{RMP}$ fields lead to a larger distortion of the plasma boundary at the midplane. However, since the field decays faster for higher-$n_{RMP}$ (which is particularly important for double null plasmas which are geometrically located further from the in-vessel coils [11]), when the resonant field at the boundary is the same, the corrugation caused by $n_{RMP} = 6$ is exacerbated.

Whilst the predictions from an ideal 3D equilibrium code such as VMEC have replicated the measured displacements in MAST [22, 32], the code assumes nested flux surfaces. Since the lobe structures near the lower divertor illustrated in figure 5 are consistent with field line reconnection, we have also applied a resistive MHD model
Figure 9. The radial position of the plasma boundary at the outboard midplane as a function of toroidal angle as predicted by VMEC when $n_{RMP} = 2, 3, 4, 6$ RMPs are applied at (left) full field and (right) scaled such that $b_{res}$ is the same at the separatrix. The displacement associated with the $n = 12$ toroidal field ripple is included here.

Figure 10. A toroidal map of the separatrix position predicted by JOREK when $n_{RMP} = 3, 4, 6$ RMPs are applied in MAST for the same in-vessel coil current, compared to the separatrix location without an applied field.

to consider the boundary displacements. Here the JOREK nonlinear MHD code [37] is run with single toroidal mode number and a non-axisymmetric field applied until it reaches saturation as a new equilibrium state [38]. It should be noted that we do not retain the higher-$n$ harmonics in these simulations (which are unstable when run with an axisymmetric equilibrium) so any non-linear coupling between the applied field and higher-$n$ modes is neglected. We consider a double-null MAST configuration making use of the double null geometry developed in reference [39]. The RMP field is ramped up over 20ms. The plasma resistivity in JOREK is assumed to be $\eta = 6 \times 10^{-6}\Omega m$ – approximately 300 times larger than in MAST – with perpendicular (parallel) viscosity of $\mu_\perp (\mu_\parallel) = 3 \times 10^{-6}(3 \times 10^{-5})\text{kg m}^{-1}\text{s}^{-1}$. Figure 10 shows the position of the separatrix as predicted by JOREK as a function of toroidal angle for MAST double-null plasmas with saturated $n_{RMP} = 3, 4, 6$ fields compared to the separatrix position without a non-axisymmetric field applied. The midplane displacement is of the order of $\pm 2.5\text{cm}$, in good agreement with the measured displacements shown in figures 7 and 8.
4. Effect of different RMP configurations on the pedestal and edge stability

4.1. Pedestal structure

When an \( n_{RMP} = 6 \) RMP with \( 0^\circ \) phase is applied, Thomson scattering measurements show a drop in density pedestal height as well as increase in both density and temperature pedestal width [33]. The three-dimensional nature of the profiles has been investigated in these \( n_{RMP} = 6 \) \( 0^\circ \) phase experiments with MAST plasmas with \( n_{RMP} = 6 \) with \( 30^\circ \) phase by comparing the pedestal profile measured by the Thomson scattering diagnostic with linear \( D_\alpha \) camera measurements in an adjacent MAST sector (ie approximately \( 30^\circ \) out of phase). ERGOS predicts that the boundary perturbation upon application of an \( n_{RMP} = 6 \) with \( 0^\circ \) phase (\( n_{RMP} = 6 \) with \( 30^\circ \) phase) RMP will be maximum (minimum) when measured by the Thomson scattering diagnostic and correspondingly minimum (maximum) when measured by the linear \( D_\alpha \) camera. The measurements of density gradient as a function of radius from these two diagnostics are shown in figure 11. Here, the \( D_\alpha \) emissivity is obtained from spatial inversion of line of sight integrated intensity \( D_\alpha \) profiles as measured by a linear camera. The edge density profile gradient may be estimated from these measurements as outlined in reference [34]. Although the absolute value of the gradient differs from the Thomson scattering gradient, the relative variation of gradient levels is found to be consistent for shots with no applied RMP. The Thomson scattering shows that the pedestal width increases and location moves outward for the \( n_{RMP} = 6 \) \( 0^\circ \) RMP, whilst the linear \( D_\alpha \) data shows that the peak gradient location does not move and the density gradient magnitude shows a slight increase. Conversely, with an \( n_{RMP} = 6 \) \( 30^\circ \) RMP field, the linear camera measures the peak gradient to decrease strongly and move outwards by \( \sim 20 \)mm whilst the Thomson scattering observes negligible change in the position of the pedestal but an increase in the density gradient. Since there is a density pedestal height drop on application of RMP, the density width is contracting to cause an increase in gradient. No change in pedestal width has been observed from examination of the profiles on the field side when RMPs are applied. This indicates toroidally varying density pedestal width, and hence pressure gradients on the low field side when non-axisymmetric fields are applied, indicating the importance of performing stability calculations based upon a three dimensional equilibrium.

This toroidal variation of the pressure gradient is present upon application of all mode numbers of RMP. Figure 12 shows the degradation of the pedestal height and increase in the pedestal width for \( n_{RMP} = 3, 4, 6 \) for plasmas with very similar \( b'_{res} \). This degradation in the pedestal density is replicated by JOREK resistive MHD modelling, where the RMPs cause enhanced particle transport and a decrease in pedestal pressure, for all \( n_{RMP} \), as seen in figure 13. In JOREK the enhanced particle transport results from both convective cells [35] and magnetic field stochasticity, thus both density and temperature are lost, contrary to the experimental observations where the temperature is largely unperturbed. The degradation of the pedestal height is similar for all \( n_{RMP} \), and in all cases, the pedestal pressure can be largely recovered by refuelling the plasma.
Figure 11. The radial profile of the electron density gradient as measured by (left) the Thomson scattering diagnostic and (right) the $D_\alpha$ camera in MAST with and without an $n_{RMP} = 6$ RMP applied.

Figure 12. The electron density and temperature measured by the Thomson scattering at the end of the ELM cycle in a shot without RMPs compared to the profiles with $n_{RMP} = 3, 4, 6$ applied averaged over multiple ELMs. The time-windows over which the with and without RMP cases are averaged is different for each $n_{RMP}$, explaining why the profiles without RMPs change slightly.

to counter the enhanced particle transport incurred by the RMP fields [36].

The evolution of the pedestal in mid-collisionality plasmas ($\nu_e^* \in [0.8, 1.5]$) such as those presented here has been attributed to mediation by kinetic ballooning modes limiting the pedestal gradient until the pedestal width increases to the point at which finite-$n$ ballooning modes become unstable [40]. Infinite-$n$ ballooning stability has been
Figure 13. The pedestal density and pressure height as calculated by JOREK for a 3D equilibrium when $n_{RMP} = 3, 4, 6$ RMPs are applied.

Compared to gyrokinetic analysis for MAST plasmas without RMPs and shown to be a good proxy for kinetic ballooning mode stability [40]. Here we use the VMEC code [41] to generate non-axisymmetric equilibria for MAST plasmas with $n_{RMP} = 2, 3, 4, 6$ applied and then find the $n = \infty$ ballooning mode growth rate using the COBRA code [42] to compare to the pedestal structure shown in figure 12. Figure 14 shows the $n = \infty$ ballooning mode growth rate – as a proxy for kinetic ballooning mode stability – across the radial extent of the pedestal. Here, the growth rate is shown for a toroidal position where instability is exacerbated, though in a different toroidal phase the $n = \infty$ growth rate is decreased relative to the case without RMPs [54]. The non-axisymmetric perturbation of the pedestal exacerbates kinetic ballooning mode instability, widening the region over which infinite-$n$ modes are unstable and enhancing the mode growth rate. Assuming infinite-$n$ modes remain a good proxy for kinetic ballooning modes, this suggests that particle transport, assuming a crude gyro-Bohm diffusivity scaling like $\gamma/k^2$ (where $\gamma$ is the growth rate and $k$ is the perpendicular wavenumber), is enhanced, consequently limiting the pressure gradient to a lower value. This is consistent with the pedestal width increasing locally in the most unstable toroidal position before the finite-$n$ peeling-ballooning limit is reached after the application of RMPs, as observed empirically in figure 11. The region over which the infinite-$n$ ballooning modes become unstable, and the enhancement in their growth rate, is similar for all RMP configurations, consistent with the pedestal structure being similar for all $n_{RMP}$. The pedestal pressure must be reduced by 30% for the infinite-$n$ ballooning mode growth rate in the toroidal position of greatest instability to equal that in the case with no RMPs [54].
5. Other effects of different RMP configurations on the plasma

5.1. Toroidal plasma rotation braking

The toroidal rotation of the plasma is observed to decrease across the whole plasma minor radius when the RMPs are applied to single null discharges, such as those shown in figure 1. Braking occurs for all RMP configurations, though it is so severe for $n_{RMP} = 2, 3$ it causes a back transition before the current in the in-vessel coils reaches its maximum value (at least for the $q_{95}$ and proximity to the in-vessel coils of the plasma studied here). Figure 15 shows the toroidal rotation measured by charge exchange recombination spectroscopy at a radial position just inside the top of the pedestal as a function of time. Also shown is the waveform of the in-vessel coil currents, illustrating that different maximum field amplitudes are applied since the braking is so severe for low-$n_{RMP}$ as to cause loss of H-mode. The plasma rotation decreases slightly even in the absence of applied RMPs, likely to be due to the torques associated with intrinsic error fields [43]. The deceleration is approximately the same for all RMP configurations, though the saturated level depends on $n_{RMP}$. The braking is strongest for $n_{RMP} = 2$, noting that the applied field is only a third as large as other configurations to preclude a back-transition. $n_{RMP} = 3$ induces the next strongest braking torque, with $n_{RMP} = 4, 6$ producing a similar braking and saturating at finite rotation.

Figure 16 shows the time evolution of the plasma rotation at different rational surfaces in these MAST single null plasmas. In all cases there is global plasma braking across the whole of the minor radius, with the braking being strongest for lower-$n_{RMP}$ fields.

The effect of RMPs on the plasma flow has been simulated using the quasi-linear MARS-Q code [44]. MARS-Q solves the full MHD, single-fluid equations to find the plasma response in full toroidal geometry. These perturbed MHD equations are then
Figure 15. The rotation frequency as measured by the charge exchange spectroscopy in MAST at $\psi_N = 0.3$ as a function of time. Also shown is the current in the in-vessel coils, which is lower amplitude for low-$n_{RMP}$ to avoid plasma locking causing an H-L transition.

Figure 16. The rotation frequency as measured by the charge exchange spectroscopy in MAST at different rational surfaces when $n_{RMP} = 2, 3, 4, 6$ RMPs are applied in single-null plasmas shown in figure 15.
coupled to the momentum balance equation for the change in the toroidal flow of the plasma and solved using a semi-implicit adaptive time-stepping scheme non-linearly in time. The model incorporates both the NTV torque [47] and the $j \times B$ torque [46] with both resonant and non-resonant contributions included. To simulate the MAST plasmas shown in figure 16, a resistive plasma model is assumed with magnetic Lundquist number $S_0 = 3.5 \times 10^7$ at the magnetic axis, with a profile given by $S \sim T_e^{3/2}$, so $S_{edge} \approx 10^6$. The non-axisymmetric field is ramped up over 20ms, as done experimentally, and the intrinsic error field is included for the $n_{RMP} = 2$ case. It is evident that a similar braking is replicated near the plasma edge for the higher-$m$ rational surfaces. Whilst the deceleration is similar in the core plasma, the saturated rotation level is higher numerically than observed in MAST.

The rotation braking predicted by Mars-Q in these plasmas is less than in other single null MAST discharges – for instance, those reported in higher-current plasmas in references [48, 11, 44] – thought to be due to the initial rotation profile. These simulations begin with an initial rotation profile with slower edge velocity, presumably due to enhanced fast ion losses at lower current giving rise to counter-current fast particle driven $j \times B$ torques [45]. The braking associated with the RMPs is equally due to the NTV and $j \times B$ torques, whereas the braking in the case with a faster edge rotation is dominated by $j \times B$ torque. Increasing the momentum diffusivity does not lead to an increase in the core braking, with the simulations proving more sensitive to the initial rotation profile. Nonetheless, the edge rotation braking predicted by Mars-Q agrees very well, both in amplitude and deceleration, with observations from MAST.

5.2. Degradation of access to H-mode

Dedicated experiments have been performed to assess the implications of applying RMPs in $n_{RMP} = 2, 3, 4, 6$ configurations on the power required to access high confinement
Figure 18. Timetraces of the $D_\alpha$ emission for MAST plasmas with (b) no RMP and (c-f) with $n_{RMP} = 6, 4, 3, 2$ respectively. The RMP waveform is also shown and $P_{NB1} = 1.5$MW.

The RMPs are applied before the time at which the L-H transition occurs in a discharge with no applied field. The discharge shown in figure 18 has 1.4kA of current in each of the RMP coils and 1.5MW of beam power. Applying 0.5MW of beam power is sufficient to cause an L-H transition when no RMPs are applied, hence these discharges are significantly above the no-RMP L-H transition power threshold. The shots are in density feedback, maintaining the density at the transition time taken from the no applied RMP discharge. Without this feedback, the particle pumpout caused by the RMPs prevents H-mode access due to changes in the density dependent power threshold. The application of $n_{RMP} = 3$ and $n_{RMP} = 4$ RMP in figure 18 cause a short delay of the L-H transition time of approximately one energy confinement time. The application of $n_{RMP} = 6$ also results in a delayed transition, although the delay is significantly longer. The magnitude of the applied RMP field in this case is sufficient to cause an approximate doubling of the ELM frequency in the $n_{RMP} = 3, 4$ and 6 cases. At the same RMP coil current level, the application of $n_{RMP} = 2$ RMPs causes a complete suppression of the transition. These results suggest that the power required to access H-mode is non-monotonic with $n_{RMP}$ and is optimal for an RMP configuration with $n_{RMP} = 3$ or 4 in MAST. It is not yet understood why the power required to access H-mode is lowest for these configurations. The density and temperature profiles for the different RMPs show very little difference before the L-H transition, though there are demonstrable differences in the plasma rotation, suggesting that a modification of the radial electric field by the RMPs could cause the variation in the L-H power threshold.
5.3. Enhanced fast ion losses

The effects of RMPs on fast particle confinement in MAST have been modelled using the orbit-following codes LOCUST [49] using either an equilibrium with plasma response included from the MARS-F code [50], or using a vacuum approximation from ERGOS. Simulations with a large number of test particles indicate a significant increase in beam power loss in the case of a single null MAST plasma equilibrium, from (5-10)% in the axisymmetric limit to as much as 50% when RMPs are applied, depending on the model used for the first wall and whether the plasma response to the RMPs is taken into account. LOCUST results indicate that the losses have a non-monotonic dependence on $n_{RMP}$, with the fewest losses occurring for $n_{RMP} = 4$. In all cases studied, at least for these high-$\beta_N$ single null MAST plasmas, the plasma response increased the power loss, by up to a factor of two.

6. Discussion and Conclusions

The MAST experiments and numerical modelling presented here has focussed on the effects of RMPs on various plasma parameters when different RMP configurations are applied. The in-vessel coil set planned for ITER has 27 coils with individual power supplies, meaning that there is significant flexibility in the system to optimise the periodicity of the applied field, with $n_{RMP} = 3$ or 4 the anticipated configurations. Since the primary purpose of the in-vessel coils is to mitigate the ELM heat flux sufficiently to ensure an adequate lifetime of plasma facing components, if there were an RMP configuration which maximised the peak heat flux reduction, this would be the major consideration.

In MAST, ELM mitigation has been achieved across a wide range of pedestal collisionalities, including those for which other experiments have observed ELM suppression [6, 7]. For these plasmas with mitigated type-I ELMs, the peak heat flux scales with the ELM frequency, with the different RMP configurations all exhibiting this linear relationship. Consequently, a configuration which maximises the ELM frequency will mitigate the peak heat flux most effectively. Since various RMP periodicities are predicted to meet the empirical conditions for ELM suppression in ITER [16], it is likely that, notwithstanding the uncertainties in the understanding of RMP ELM suppression which negate reliable extrapolation, ELM mitigation will be possible. Indeed, ELM mitigation has been achieved in all machines, and over a much wider parameter space than ELM suppression [51]. Figure 3 shows that above a threshold value, the ELM frequency, and hence the reduction in peak heat flux, scales linearly with the applied field. This relationship between the applied field amplitude and the mitigated ELM frequency is common across all machines [51] suggesting that the reduction in peak heat flux will be comparable for all $n_{RMP}$ in ITER since the $b_{res}$ threshold value for ELM mitigation extrapolated from current devices is considerably lower than the achievable value in ITER. Consequently, the choice of optimal RMP configuration will
be determined by other factors; a summary of the optimal configurations for these other physical metrics in MAST is given in table 1.

<table>
<thead>
<tr>
<th>Criterion</th>
<th>$n_{RMP} = 2$</th>
<th>$n_{RMP} = 3$</th>
<th>$n_{RMP} = 4$</th>
<th>$n_{RMP} = 6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Locked mode threshold</td>
<td>Lowest</td>
<td>Lowest</td>
<td>Highest</td>
<td></td>
</tr>
<tr>
<td>Reduction in $q_{peak}$</td>
<td></td>
<td>Lowest</td>
<td></td>
<td>Highest</td>
</tr>
<tr>
<td>Critical $b'_{res}$ to mitigate ELMs</td>
<td>No data</td>
<td>No data</td>
<td>Lowest</td>
<td></td>
</tr>
<tr>
<td>Power for H-mode</td>
<td>Highest</td>
<td>Highest</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fast ion loss</td>
<td>No data</td>
<td>Most (plasma)</td>
<td>Most (vacuum)</td>
<td></td>
</tr>
<tr>
<td>Midplane displacement</td>
<td>No data</td>
<td>Least</td>
<td>Most</td>
<td></td>
</tr>
<tr>
<td>Lobe structures</td>
<td>No data</td>
<td>Shortest</td>
<td>Longest</td>
<td></td>
</tr>
<tr>
<td>Pedestal structure</td>
<td></td>
<td>Region and growth rate scales with $b'_{res}$ in all cases</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Infinite-n stability</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rotation braking</td>
<td>Strongest</td>
<td></td>
<td></td>
<td>Weakest</td>
</tr>
</tbody>
</table>

Table 1. Summary of the merits of the various $n_{RMP}$ RMP configurations in MAST for various criteria ordered from the top to bottom, firstly in consideration of safe operation of ITER, and secondly in terms of the most critical for successful operation to $Q = 10$. Note that here the criteria on fast ion loss and H-mode access are based upon measurements made with the same coil current rather than fixed $b'_{res}$, whereas the criteria for midplane corrugation, lobe structures and rotation braking are taken from cases with the same $b'_{res}$.

It is evident that $n_{RMP} = 2$ at sufficient amplitude to mitigate the ELM heat flux has the most detrimental effect on these MAST plasmas. Conversely, RMPs with periodicity $n_{RMP} = 3, 4$ have least deleterious side-effects on the plasma when mitigating ELMs. Whilst $n_{RMP} = 4$ has a higher locked mode threshold, $n_{RMP} = 3$ has a lower critical field for increasing the ELM frequency, which means that less applied field is required to achieve the same reduction in peak heat flux. $n_{RMP} = 4$ RMPs also have the advantage of causing least degradation of fast ion confinement and the power threshold required to access H-mode. Whilst $n_{RMP} = 3$ cause less radially extended lobe structure near the X-point, they are more poloidally extended, so may interact more with the first wall.

The reasonable agreement between numerical modelling with MAST data for rotation braking, three-dimensional corrugation and lobe structures shown in this paper suggests that these models can be applied to future machines to make firm predictions for the effects of different RMP configurations in ITER; this will be the subject of future work. Indeed, based upon the validation of numerical tools with MAST data, assessments of the three dimensional midplane corrugation and rotation braking have been made for ITER. The midplane displacements are predicted, both with 3D equilibrium modelling and nonlinear resistive MHD codes, to be comparable for $n_{RMP} = 3$ and 4 fields and within the allowable limits anticipated for plasma facing component heat loading [31]. Quasi-linear MARS-Q simulations of the RMP induced
Merits of RMPs with different toroidal mode numbers

20

torques show that $n_{RMP} = 3$ and 4 RMPs produce similar edge braking, but neither are predicted to have a detrimental effect on the core plasma [52]. Finally, it is worth pointing out that vacuum calculations for ITER [16] suggest that the toroidal asymmetry of the lobe structures is far higher for an $n_{RMP} = 4$ configuration than $n_{RMP} = 3$ due to an interference of the $n_{RMP} = 4$ and $n_{RMP} = 5$ components of the spectrum, potentially resulting in an asymmetric heat loading of the ITER divertor. Furthermore, the wetted area for an $n_{RMP} = 3$ footprint is predicted to be approximately twice as large as for an $n_{RMP} = 4$ RMP, favouring the use of $n_{RMP} = 3$ non-axisymmetric fields in ITER.

Overall, non-axisymmetric fields with toroidal mode number of $n_{RMP} = 3$ or 4 are optimal for RMP ELM control in MAST since they have less detrimental effects on the power to access H-mode, fast ion confinement, distortions to the magnetic configuration and the plasma rotation, whilst still having a low threshold for an effect on the pedestal and ELMs and a higher tolerance to locked modes than low-$n_{RMP}$ RMPs. The fact that the optimal $n_{RMP}$ can be tuned depending on which ancillary plasma effect is considered most important strongly suggests that ITER should retain the capability to operate with RMPs with a range of different mode numbers and spectra to maximise the likelihood that the in-vessel coils will be able to control the ELMs to ensure an adequate lifetime of the plasma facing components and simultaneously have minimal deleterious effect on the plasma performance.

Acknowledgements

This work was part-funded by the RCUK Energy Programme [grant number EP/I501045] and by the European Union’s Horizon 2020 research and innovation programme. To obtain further information on the data and models underlying this paper please contact PublicationsManager@ccfe.ac.uk. The views and opinions expressed herein do not necessarily reflect those of the European Commission.

[13] Liang Y et al 2010 Nucl Fusion 50 025013
[16] Evans, T.,E., et al., 2013 Nucl. Fusion 53 093029
Merits of RMPs with different toroidal mode numbers

21

Single-Null Discharges on MAST”
[27] Jakubowski MW et al 2009 Nucl. Fusion 49 095013
[31] IT Chapman et al 2014 accepted Nucl Fusion “Three dimensional distortions of the tokamak plasma
boundary: Boundary displacements in the presence of resonant magnetic perturbations”
system on the three dimensional distortion of the plasma boundary when magnetic perturbations
are applied in MAST”
collisionality discharges in MAST and a comparison with ASDEX Upgrade”
[38] F Orain et al Physics of Plasmas 20 102510 (2013)
[43] YQ Liu and A Kirk sub Plasma Phys Control Fusion “Modelling error field correction experiments
in MAST”
[52] YQ Liu et al, in prep Nucl Fusion 2014 “Modelling of toroidal rotation damping in ITER due to
external 3D fields”
[54] CJ Ham et al, sub Phys Plasmas 2014 “Modelling of three dimensional equilibrium and stability
of MAST plasmas with magnetic perturbations using VMEC and COBRA”