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Comparison of BES measurements of ion-scale turbulence with direct gyrokinetic simulations of MAST L-mode plasmas

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Abstract. Observations of ion-scale ($k_y \rho_i \leq 1$) density turbulence of relative amplitude $\gtrsim 0.2\%$ are available on the Mega Amp Spherical Tokamak (MAST) using a 2D (8 radial $\times$ 4 poloidal channel) imaging Beam Emission Spectroscopy (BES) diagnostic. Spatial and temporal characteristics of this turbulence, i.e., amplitudes, correlation times, radial and perpendicular correlation lengths and apparent phase velocities of the density contours, are determined by means of correlation analysis. For a low-density, L-mode discharge with strong equilibrium flow shear exhibiting an internal transport barrier (ITB) in the ion channel, the observed turbulence characteristics are compared with synthetic density turbulence data generated from global, non-linear, gyro-kinetic simulations using the particle-in-cell (PIC) code NEMORB. This validation exercise highlights the need to include increasingly sophisticated physics, e.g., kinetic treatment of trapped electrons, equilibrium flow shear and collisions, to reproduce most of the characteristics of the observed turbulence. Even so, significant discrepancies remain: an underprediction by the simulations of the turbulence amplitude and heat flux at plasma periphery and the finding that the correlation times of the numerically simulated turbulence are typically two orders of magnitude longer than those measured in MAST. Comparison of these correlation times with various linear timescales suggests that, while the measured turbulence is strong and ‘critically balanced’, the simulated turbulence is weak.

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1. Introduction

A ‘grand challenge’ of fusion research is to develop reliable, first-principles predictive capability of plasma confinement, e.g., of the ITER device. An essential part of this process is to perform quantitative comparisons of simulation results with experimental observations and thereby assess their validity [1]. Prediction of heat and particle fluxes require non-linear simulations of the saturated state of plasma turbulence, which can be validated at one level by comparing with the results of transport analysis and, at a deeper level, with measured turbulence characteristics. An excellent overview of the methodology of such studies can be found in Ref. [2].

The micro-instabilities responsible for anomalous transport exist over a wide range of spatial scales: from electro-static ion-temperature-gradient (ITG), trapped-electron (TEM) and parallel-velocity gradient (PVG) modes (or electro-magnetic micro-tearing modes at high-$\beta$) at scales larger than the ion Larmor radius $k_y \rho_i \leq 1$ (where $k_y$ is the wavenumber perpendicular to the magnetic field, within a flux surface and $\rho_i$ is the ion Larmor radius), down to electron-temperature-gradient (ETG) modes with $k_y \rho_i \gg 1$ [3]. A wide range of diagnostic techniques is required to detect the resulting fluctuations in plasma parameters. Such verification programmes have been the focus of much research activity on conventional tokamaks, e.g., on DIII-D, which has a comprehensive set of turbulence diagnostics [2, 4, 5] and on Tore Supra [6, 7]. Studies on the NSTX spherical torus have focused on electron-scale turbulence, comparing data from a microwave scattering system with global, non-linear gyro-kinetic simulations using GYRO [8] (earlier studies focused on linear gyro-kinetic calculations [9, 10]). The availability of ion-scale density fluctuation measurements from BES [11] will facilitate future multi-scale validation studies on the NSTX device.

On DIII-D, a unique array of multi-scale, multi-field turbulence diagnostics has facilitated detailed validation studies comparing gyro-kinetic turbulence simulations with experimental data from L- and H-mode plasmas [12–14]. A variety of studies have been undertaken, e.g., of the dependence of the turbulence characteristics on $T_e/T_i$ [2] and elongation [15] and of the cross-phase between $\delta n_e$ and $\delta T_e$ fluctuations [16], thus probing the underlying physics, especially in cases where disagreement was found. Meaningful comparison of simulation results with experiment is only possible by implementing synthetic diagnostics that mimic the instrumental characteristics of the various fluctuation measurements, as was done, e.g., on DIII-D for BES and Correlation Electron Cyclotron Emission (CECE) diagnostics for measurements of $\delta n_e/n_e$ and $\delta T_e/T_e$, respectively [13, 16, 17]. In the L-mode studies, although good agreement was found between simulated and measured heat flux and fluctuation characteristics in the mid-core region ($0.4 < r/a < 0.75$, where $r/a$ denotes the normalised radius), simulations generally underpredict the heat fluxes and amplitudes by almost an order of magnitude in the peripheral region ($r/a > 0.75$) [2, 4, 13, 14, 16]. This discrepancy has been attributed to increasing importance of edge-core coupling [18] or to an inward propagation of turbulence (or ‘avalanche’) from the plasma edge [19]. More recently,
there have been concerted efforts to resolve this discrepancy, which now appears not to be ubiquitous.

No such validation exercise using global, non-linear simulations of ion-scale turbulence has as yet been performed for a spherical-tokamak plasma, whose particular characteristics make this especially interesting. Heating of the low-aspect-ratio plasma with tangentially directed neutral-beam-injection (NBI) heating results in strong equilibrium rotation in the core (toroidal Mach number $M_\phi = R\omega_\phi/v_{\text{thi}} \lesssim 0.5$, where $\omega_\phi$ is the toroidal rotation rate and $v_{\text{thi}}$ is the ion thermal velocity) and hence strong $E \times B$ shear $\gamma_E = (\epsilon/q) d(R\omega_\phi)/dr$, where $\epsilon = r/R$ and $q$ the safety factor, which can be strong enough to stabilise ion-scale turbulence. The low aspect ratio also results in a large trapped particle fraction ($f_t \sim \epsilon^{1/2}$), enhancing the drive for Trapped-Electron Mode (TEM) micro-instabilities, although, as will be seen, collisions can reduce the trapped particle drive substantially in MAST plasmas. Finally, the low toroidal magnetic field $B_\phi$ results in larger values of the ion Larmor radius normalised to the plasma radius, $\rho_* \equiv \rho_i/a$. The assumption of constant gradient scale lengths in local gyro-kinetic simulations is questionable when gradients vary appreciably over the simulation domain, which typically has to be several $10 \times \rho_i$ in radial extent to ensure convergence. Therefore, as discussed in Ref. [20], for current-generation ST plasmas in which $\rho_* \sim 1/50$, global codes may be required to perform meaningful non-linear simulations of the ion-scale turbulence.

Here we present the first validation exercise for a MAST plasma comparing the characteristics of ion-scale turbulence measured using a 2D BES turbulence imaging system [21] with synthetic data produced from non-linear simulations performed using the global particle-in-cell (PIC) code NEMORB [22]. The comparison is performed for a low-density, L-mode plasma exhibiting an internal transport barrier (ITB), similar to those used for earlier studies of ITB formation and dynamics [23, 24], for which linear gyro-kinetic stability calculations have been performed using the local code GS2 [25]. This equilibrium configuration was also used as the basis for the earlier global, non-linear simulations presented in Ref. [20]. An L-mode plasma is ideal for these studies because, in the peripheral region, the equilibrium flow shear is too weak to stabilise the ITG turbulence fully.

The MAST device is a medium-sized, low-aspect-ratio tokamak (aspect ratio $A = R/a \sim 1.3$, plasma current $I_p \leq 1.2$ MA, toroidal field $B_\phi \leq 0.58$ T at 0.7 m), which is equipped with tangentially directed NBI heating (injected power $P_{\text{NBI}} \leq 3.8$ MW at $\sim 70$ keV $D^0$ injection energy). Studies of transport in MAST are facilitated by the availability of high-resolution kinetic profile data from a suite of advanced diagnostics (see §3) and an integrated analysis chain ($MC^3$) to prepare this data for transport analysis using TRANSP [26]. The BES turbulence imaging system on MAST has sufficient signal-to-noise ratio (SNR) to detect density fluctuations with relative magnitude $\delta n_e/n_e \gtrsim 0.2\%$ if appropriate correlation analysis is used. Calculations of the three-dimensional spatial response and sensitivity of the BES system, accounting for relevant physical effects were performed in Ref. [27] as described in §3.1. This is
quantified in terms of two-dimensional point-spread functions (PSFs), which are used here to generate synthetic BES data from the gyro-kinetic simulation data. Previous results of an analysis of the BES data [28] to elicit the structure and dynamics of the ion-scale turbulence [29] and the dependence of the ion temperature gradient on the magnetic configuration, rotational shear and heat flux [30] are pertinent to our discussion of the results presented here.

The remainder of the paper is structured as follows. The results of gyro-kinetic simulations of MAST L-mode equilibria are presented in §2, with a summary of results of previous linear calculations in §2.1 and the non-linear simulations in §2.2. Our experimental observations of the ion-scale turbulence are explained in §3, with a brief description of the capabilities of the BES system in §3.1 and an explanation of the correlation analysis used to determine the turbulence characteristics in §3.2. The method used for the generation of synthetic data is then explained in §3.3. The observed and simulated turbulence characteristics are then compared in §4. Finally, our conclusions are presented in §5.

2. Gyro-kinetic simulations of an L-mode discharge

The validation study presented here is for an L-mode discharge formed using a scenario with early NBI heating during the current ramp, which slows current penetration, giving rise to strong toroidal rotation and negative magnetic shear (\( \dot{s} = (r/q)(dq/dr) \)) in the plasma core. Previous linear-stability calculations for such a discharge [24] revealed the peripheral region, where the flow shear is much weaker, to be unstable to ITG modes. Therefore, such discharges hosting ITG turbulence are ideal candidates on which to validate non-linear, ion-scale turbulence simulations.

Kinetic profiles of the equilibrium used for this study are shown in Fig. 1. They are taken from MAST L-mode discharge #27268 at 0.25 s, the parameters of which are: plasma current \( I_p = 800 \text{kA} \), toroidal field \( B_\phi \sim 0.58 \text{T} \) at \( R_m = 0.7 \text{m} \), co-injected NBI heating of \( P_{NB} = 3.3 \text{MW} \), line-average density \( \bar{n}_e \leq 2.3 \times 10^{19} \text{m}^{-3} \), with \( T_i \leq 2.2 \text{keV} \), \( T_e \leq 1.5 \text{keV} \) and \( \omega_\phi = 2.0 \times 10^5 \text{rad/s} \), corresponding to Mach number \( M_\phi \leq 0.5 \). The foot of the ITB in the ion thermal and momentum channels is located at the normalised radius \( \rho_n \equiv \psi_\lambda^{1/2} \sim 0.5 \) (where \( \psi_\lambda \) is the normalised poloidal flux) in the region where \( \dot{s} \leq 0 \). Note that the temperature and rotation profiles of the bulk \( D^+ \) ions are calculated from measurements on the \( C^{6+} \) impurities using the NCLASS [31] neo-classical package within the TRANSP code.

Linear-stability calculations were performed using both GS2 [24] and NEMORB [20] for a similar equilibrium from an earlier MAST discharge (#22087 at 0.25 s), albeit with somewhat higher plasma current of 0.88 MA, which was used for a study of ITB formation and evolution [24]. The kinetic profiles from this equilibrium are very similar to those of the equilibrium studied here, so the results of these linear calculations are relevant and briefly summarised in §2.1. This equilibrium was also used as the basis for first global, non-linear simulations [20] using NEMORB and some of the relevant results
Figure 1: Equilibrium profiles for discharge #27268 at 0.25 s: (a) ion $T_i$ (black) and electron $T_e$ (red) temperatures, (b) electron density $n_e$, with the edge-shielding density (dashed, see §2.2.3), (c) toroidal rotation rate $\omega_{\phi}$ for the $C^{6+}$ ions (solid) and $D^+$ ions (dashed) and (d) the $q$-profile from EFIT. The raw data from the CXRS and TS systems is superposed where appropriate.

are also summarised in §2.2.

2.1. Linear stability calculations of #22807 at 0.25 s.

As presented in Ref. [24], local linear-stability calculations with kinetic electrons were performed using GS2 for the equilibrium from discharge #22807 at the time of peak ITB strength at 0.25 s at three locations: inside the ITB, just outside $q_{min}$ and in the plasma periphery. At the innermost of these surfaces (at $\Phi^1/2_N = 0.3$, where $\Phi_N$ is the normalised toroidal flux, corresponding to $\rho_n \sim 0.36$) all modes at electron and ion scales were found to be stable in both electrostatic and electromagnetic calculations. At mid-radius ($\Phi^1/2_N = 0.52$, corresponding to $\rho_n = 0.67$), in the absence of flow shear, the relatively low collisionality results in appreciable trapped-electron drive and TEM modes are unstable in the intermediate wave-number range ($1 < k_\perp \rho_i < 10$) – these are not completely stabilised by the flow shear. (In contrast, in calculations with adiabatic electrons, ITG modes that are unstable without flow shear are completely stabilised when the flow shear is included). At the outermost surface ($\Phi^1/2_N = 0.7$, corresponding to $\rho_n = 0.87$), the TEM modes are stable because the plasma is more collisional, while strongly growing ITG modes are not stabilised by the flow shear, which is weak at this radius.

To assess the importance of non-local effects, the results of linear stability calculations with NEMORB were compared with those from local, linear GS2 calculations (with and without collisions) [20]. Linear, electrostatic calculations were made with NEMORB (for the equilibrium from #22807 at 0.25 s) with adiabatic and kinetic electrons, both with and without flow shear [32], but without collisions. As in the GS2 calculations, the core was found to be stable to ITG modes, while the most unstable region was found to be $0.6 < \rho_n < 0.8$. By varying the normalised ion Larmor
radius $\rho_*$ in the NEMORB simulations, it was found that global effects significantly reduce the growth rates, starting at $\rho_*$ just below the experimental value of 0.018.

2.2. Global, non-linear simulations with NEMORB

2.2.1. Previous simulations of #22807 at 0.25 s. The results of global, electrostatic, non-linear simulations with NEMORB of the equilibrium from discharge #22807 at 0.25 s were presented in Ref. [20] and the reader is referred to this article for details. In the simulations with adiabatic electrons but without sheared flow, the turbulence was found to spread from the linearly unstable region at $\rho_n \sim 0.7$ into the stable region down to $\rho_n \leq 0.4$ during the non-linear phase. The ion heat flux was found to be very low $Q_{i}^{\text{sim}} \leq 0.01 \text{MWm}^{-2}$, which is below the neo-classical level. Increasing the normalised, inverse ion-temperature-gradient scale length $R/L_{T_i} = R \nabla T_i / T_i$ by 30% increased the heat flux considerably, indicating that the gradient is close to the non-linear threshold for ITG turbulence. In simulations with kinetic electrons but without sheared flow, the heat flux increased to $Q_{i}^{\text{sim}} \leq 1.2 \text{MWm}^{-2}$, which is far above the experimental level, $Q_{i}^{\text{exp}} \sim 0.02 \text{MWm}^{-2}$. Again, the turbulence spread into the linearly stable core region. Including equilibrium flow suppressed the turbulence in the region with strong shear ($\rho_n \leq 0.45$) and modestly reduced the peak heat flux. Results of runs incorporating electron collisions, which had been shown to reduce the linear drive due to trapped electrons [33], were not reported in Ref. [20] (simulations with collisions have however been performed for the more recent equilibrium discussed in §2.2.2).

2.2.2. Non-linear simulations of #27268 at 0.25 s. In order to be able to compare our BES measurements with the non-linear simulations, further simulations have been carried out for an equilibrium from which the measurements are available, i.e., that corresponding to the kinetic profiles shown in Fig. 1. Electrostatic simulations are available for five cases, distinguished by whether they were run with adiabatic (AE) or kinetic electrons (KE), with or without ion-electron and electron-electron collisions and with or without equilibrium toroidal flow. The various combinations for the five runs (I-V) are summarised in Table 1, where the start time and duration of the non-linear phase used to generate the synthetic data are also stated. As discussed in §3.2 below, these periods are considered the minimum required to extract reasonable estimates of the turbulence characteristics. The evolution of the maximum heat flux $Q_{i}^{\text{max}}$ over the radial profile during each of the non-linear runs is shown in Fig. 2.

2.2.3. Details of numerical set-up. In NEMORB, the heat source is adjusted until the kinetic profiles match those prescribed and the resulting heat flux is a prediction which can be compared with the experimental value, although, as is discussed in §4.1, a precise match cannot be expected. In order to maintain the signal-to-noise ratio $\geq 10$ for the duration of the simulation, $2 \times 10^8$ markers were used on an $N_x \times N_y \times N_\phi = 100 \times 512 \times 256$ grid, for the simulations with collisions the number of markers was doubled. For the
Comparison of turbulence measurements with direct gyrokinetic simulations

Figure 2: The evolution of the maximum heat flux $Q_{i}^{\text{max}}$ during each of the non-linear NEMORB simulations listed in Table 1 where the plot symbols for the five cases are defined. The horizontal bars indicate the time periods used for the synthetic data generation.

Simulations with kinetic electrons, the trapped-electron response is treated kinetically while that of the passing electrons is treated adiabatically. The reader is referred to Ref. [20] for further numerical details.

In order to ensure numerical stability, a boundary condition is adopted in NEMORB which forces both the density and potential perturbations at $\rho_n = 1$ to be zero, $\delta n_e/n_e = \varphi = 0$. Markers near the plasma boundary carry some density which is zeroed just before the Poisson equation is solved. Moreover, if a marker or one of its associated gyro-points lies outside the plasma, it is not taken into account. Therefore, the quasi-neutrality condition is violated near the boundary and charge accumulation can occur, leading to a numerical sheath region with spurious electric fields. In order to circumvent this problem, the Poisson equation is solved with an artificial ‘shielding’ density in addition to the prescribed density near the boundary. The shielding density,

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Table 1: Summary of NEMORB simulation cases, showing the symbols used in Figs. 2-8 below.
as shown in Fig. 1(b), peaks at the plasma boundary, decaying exponentially over a scale length $\Delta \rho_n = 0.02$. Sensitivity studies have been carried out to determine the effect of changing the scale length of the shielding density, whereby it was found that increasing this from $\Delta \rho_n = 0.02$ to 0.03 has little effect on the amplitude profile, which in all simulated cases peaks deeper inside the plasma (at or inside $\rho_n \sim 0.9$).

As discussed in §3.3, generation of the synthetic BES data requires output of the simulated density fluctuations $\delta n_e/n_e(R, Z, t)$ as a function of time over a 2D (radial $R \times$ vertical $Z$) grid encompassing the spatial extent of the measurements. For simulations with adiabatic electrons, the density fluctuations are calculated from the perturbed potential $\varphi$ by assuming a Boltzmann response for the electrons $\delta n_e/n_e \approx e\varphi/T_e$. For the simulations with kinetic electrons, in order to decrease the computational time, only the trapped electrons are treated kinetically and, in this case, $\delta n_e/n_e$ is calculated from the full distribution function assuming a Boltzmann response for the passing electrons. An example of synthetic data from a non-linear simulation is shown in Fig. 3.

3. Experimental measurements

The ion-scale density fluctuations in MAST plasmas are measured using a 2D imaging BES diagnostic [21], which is briefly described in §3.1. The characteristics of the turbulence (amplitude, correlation times and lengths and apparent poloidal phase velocity of the density contours) are determined using the correlation analysis described in §3.2, applied to short ($\sim 5$ ms) time series corresponding to the equilibrium used for the gyro-kinetic simulations. Care was taken to choose a quiescent time period free of the fast-ion-driven MHD events, with frequencies in the range $20 - 200$ kHz, which are frequent in NBI heated discharges on MAST. Other data required for our analysis are obtained from the high-resolution profile diagnostics available on MAST: the magnetic pitch angle ($\alpha = \tan^{-1}(B_\theta/B_\phi)$, where $B_\theta$ and $B_\phi$ are the poloidal and toroidal components of the magnetic field) is measured by a 32-channel Motional Stark Effect (MSE) diagnostic [34]; ion temperature $T_i$ and toroidal flow velocity $U_\phi = R\omega_\phi$ are obtained from multi-channel, Charge-Exchange-Recombination Spectroscopy (CXRS) measurements on C$^{+6}$ impurity ions with spatial resolution of $\sim 1$ cm [35]; and electron temperatures $T_e$ and densities $n_e$ from a 132-channel NdYAG Thomson Scattering system with comparable spatial resolution [36]. The components of the magnetic field vector are obtained from MSE-constrained EFIT equilibrium reconstructions [37].

3.1. BES turbulence measurements

The ion-scale density fluctuations are measured using a 2D imaging BES diagnostic [21]. This system has a 2D Avalanche Photo-diode Detector (APD) in the form of an 8 radial $\times$ 4 poloidal channel array, which is directly imaged (rather than using optical fibres) onto the heating neutral beam with a resulting nominal spatial resolution of $\sim 2$ cm

$\dagger$ Here the radial coordinate $R$ is defined relative to the symmetry axis of the tokamak.
radially and poloidally. The Doppler-shifted $D_\alpha$ emission from the beam is selected using a band-pass interference filter. The incident light on the APD sensors with a photon flux of $\lesssim 2 \times 10^{11}$ s$^{-1}$ is detected with a SNR $\lesssim 300$ at 2 MHz digitisation rate simultaneously for all channels. If appropriate correlation analysis is used [29], the system is able to detect density fluctuations with relative amplitude $\delta n_e/n_e \gtrsim 0.2\%$ at wave numbers $|k| \leq 1.6$ cm$^{-1}$ and at frequencies up to the Nyquist frequency of 1 MHz.

The actual spatial resolution is degraded below that determined by the imaging properties of the optical system, both due to geometrical effects caused by the finite depth of the line of sight through the beam ($\sim 20$ cm), together with field-line curvature and any mismatch between the $B$-field direction and the line of sight, which is optimal when viewing at mid-radius ($R_v \sim 1.2$ m). The finite lifetime of the excited deuterium atoms ($3 - 10$ ns) also causes the $D_\alpha$ emission due to instantaneous excitation to be spatially de-localised by $\sim 1 - 3$ cm in the direction of beam propagation. The spatial response of the BES system is determined using a numerical simulation code [27], which takes account of these effects in terms of 2D point-spread functions, as discussed in §3.3. This spatial response depends on the particular magnetic equilibrium, beam parameters, viewing location and plasma profiles and so has to be calculated explicitly for each measurement.

The $D_\alpha$ emissivity of the beam is proportional to the beam density $n_0$, the plasma density $n_e$ at the observed location and to the relative population of the $n = 3$ excited state, which is determined by a collisional-radiative balance between excitation and de-excitation of the beam atoms by collisions with the plasma ions and electrons and by radiative decay. Neglecting any low-frequency (a few kHz) fluctuations in the beam density, the relative density fluctuation level can be determined from the intensity fluctuations using the relation $\delta n_e/n_e = (1/B)(\delta I/I)$, where $\delta n_e$ ($\delta I$) and $n_e$ ($I$) are the fluctuating and mean components of the plasma density (intensity of emission), respectively [38]. The differential excitation rate $B$ is obtained from the data in Ref. [39], where it is calculated using an appropriate collisional-radiative model. This parameter is a weak function of density (decreasing from 0.93 to 0.27 over the density ranging from $n_e = 10^{18}$ to $10^{20}$ m$^{-3}$) and a very weak function of temperature.

3.2. Correlation analysis

The statistical characteristics of the fluctuations are determined using correlation analysis techniques very similar to those used in Ref. [29], to which the reader is referred for further details. The data is first band-pass filtered over the frequency interval $20 - 200$ kHz to reject high-frequency noise and the beam fluctuations at a few kHz, which otherwise disturb the correlation analysis [40]. A matrix of spatio-temporal correlation functions is then calculated according to:

$$C(x, Z, \Delta x, \Delta Z, \Delta t) = \frac{\langle \delta I(x, Z, t) \delta I(x + \Delta x, Z + \Delta Z, t + \Delta t) \rangle}{\sqrt{\langle \delta I^2(x, Z, t) \rangle \langle \delta I^2(x + \Delta x, Z + \Delta Z, t + \Delta t) \rangle}},$$  

(1)
where \( x, Z \) and \( t \) denote the radial and vertical (poloidal) positions and time, respectively, \( \Delta x \) and \( \Delta Z \) are the radial and poloidal channel separations, \( \Delta t \) is the time lag and \( \langle \cdot \rangle \) denotes a time average over a period of \( \sim 5\,\text{ms} \) for the experimental data and over the periods given in Table 1 for the simulated data. Note that we use the notation \((x,y)\) for the radial and perpendicular directions relative to the flux surfaces, which for the case of the up/down symmetric, double-null diverted (DND) equilibria considered here are aligned with the \((R,Z)\) directions at the horizontal mid-plane.

The auto-covariances \( A(x, Z, \Delta t) = \langle \delta I(x, Z, t) \delta I(x, Z, t + \Delta t) \rangle \) at \( \Delta x = \Delta Z = 0 \), contain not only a component due to the true density fluctuations but also photon and electronic noise. The density fluctuation level at each radial location can be obtained from the (noise-subtracted) auto-covariance functions according to: \( \delta n_e/n_e (x) = \langle A(x, Z, Z) - A_N(x, Z, Z)/I(x, Z) \rangle \), where \( A_N \) is the auto-covariance of a signal from a calibration source containing only noise, determined at all 32 locations, then averaged (as denoted by \( \langle \cdot \rangle \)) over the four poloidally separated channels at the same radial location.

The poloidal correlation length \( \ell_Z \) is estimated using data from the four poloidally spaced channels at each radial location by fitting the averaged cross-correlations (over channels with the same \( \Delta Z \) at each radial location \( x \)), \( \bar{C}(x, \Delta x = 0, \Delta Z, \Delta t = 0) \) to the function \( f_Z(\Delta Z) = p_Z + (1 - p_Z) \cos [2\pi \Delta Z/\ell_Z] \exp [-|\Delta Z|/\ell_Z] \), where \( p_Z \) and \( \ell_Z \) are the two fit parameters. The parameter \( p_Z \) effectively subtracts any spatially constant component, usually due to large-scale, global MHD modes. The field-perpendicular correlation length on a given flux surface is then \( \ell_y = \ell_Z \cos \alpha \), where \( \alpha = \tan^{-1}(B_\theta/B_\phi) \) is the pitch angle of the magnetic field. In contrast to the poloidal correlation functions, the radial correlation functions have a monotonically decaying rather than a wave-like structure. Radial correlation lengths \( \ell_x \) are therefore obtained by fitting the averaged radial cross-correlations (over the four poloidal channels at each radial location \( x \)), \( \bar{C}(x, \Delta x, \Delta Z = 0, \Delta t = 0) \) to the function \( f_x(\Delta x) = p_x + (1 - p_x) \exp [-|\Delta x/\ell_x|] \), where the fit parameters are \( \ell_x \) and the constant \( p_x \), again serves to subtract any spatially constant component.

The correlation time \( \tau_c \) is estimated from the temporal cross-correlation functions \( C(\Delta x = 0, \Delta Z, \Delta t) \) between the four poloidally separated channels at each radial location, by finding for a particular \( \Delta Z \) the time delay \( \Delta t_{\text{peak}}(\Delta Z) \) at which the correlation function has its maximum. The peak values of the correlation functions \( C_{\text{peak}}(\Delta t_{\text{peak}}(\Delta Z)) \) are then fitted with the function \( f_{\Delta t}(\Delta t_{\text{peak}}(\Delta Z)) = \exp [-|\Delta t_{\text{peak}}(\Delta Z)|/\tau_c] \) to determine \( \tau_c \), where it is implicitly assumed that any influence of the finite parallel correlation length \( \ell_{\parallel} \) on \( \tau_c \) can be ignored.

The apparent phase velocity \( U_Z^{\text{BES}} \) of the fluctuations is determined at each radial location using a cross-correlation time delay (CCTD) technique [41], namely it is calculated by making a linear fit to \( \Delta Z(\Delta t_{\text{peak}}) \). It is found that the observed velocity is dominated by the apparent poloidal motion of field-aligned, elongated \( (\ell_{\parallel}/\ell_{x,y} \gg 1) \) eddies through a poloidal plane due to the dominant toroidal rotation of the plasma: as discussed in Ref. [40], it can be shown that \( U_Z^{\text{BES}} \approx -U_\phi \tan \alpha + U_Z \), where the toroidal
velocity \( U_\phi \gg U_Z \). This is because any poloidal flows are strongly damped \([42]\), leaving \( U_Z \) of the order of the diamagnetic velocity \( U^{\text{dia.i}} \sim \rho_* v_{\text{thi}} \).

This type of analysis, characterising the turbulence in terms of a few scalar parameters \((\delta n_e/n_e, \ell_x, \ell_y \text{ and } \tau_c)\) determined from the ensemble-averaged correlation functions, effectively yields these parameters averaged over all density fluctuations at spatial scales larger than the instrumental resolution \((k_\perp \leq 1.6 \text{ cm}^{-1})\), weighted by the RMS value of the density fluctuations \(\langle (\delta n_e/n_e)^2 \rangle^{1/2} \). These parameters are therefore representative of the fluctuations at the dominant ‘outer’, energy-containing scale of the turbulence \([43]\). In order to calculate meaningful ensemble averages, the time averaging must be done over many correlation times and wave periods, i.e., over a sample of duration \(\Delta t_{\text{avg}} \gg 2\pi/\omega_*\), where \(\omega_* \sim v_{\text{thi}} \rho_i/(\ell_y L_T)\) is the ion-diamagnetic frequency and \(L_T \equiv (\nabla T_i/T_i)^{-1}\) is the scale length of the ion temperature gradient. Taking the fiducial value \(\omega_*/2\pi \sim 10 \text{ kHz}\), sample periods of order 1 ms are required.

3.3. Synthetic BES data

For the comparisons between simulations and measurements to be meaningful, ‘synthetic’ data, analogous to the real BES data, is generated from the simulated density fluctuations and analysed using the same correlation techniques as those used for the experimental measurements. The synthetic data is generated using a synthetic ‘diagnostic’, which mimics the physical characteristics of the measurement technique. The effect of applying this processing to the simulated data is firstly to spatially average over fluctuations at smaller scales than that of the instrumental resolution and secondly to impose a lower limit to the fluctuation amplitude below which the signal is dominated by noise.

In order to generate the synthetic data, the characteristics of the BES system have to be quantified in terms of its spatial resolution, sensitivity and noise properties. These are determined using a numerical simulation, described in more detail in Ref. \([27]\), which accounts for the beam absorption and emission, magnetic field and viewing geometry. Assuming that the turbulent eddies are highly elongated along the field, \(\ell_\parallel/\ell_y \gg 1\), the spatial response of each channel \((i, j)\) can be quantified in terms of 2D point-spread functions (PSFs), \(\varphi_{ij}(R,Z)\), which are effectively cross-sections for the excitation of the \(D_\alpha\) beam emission per unit area of the field of view at the focal plane at the beam.

The PSFs depend on the beam properties, magnetic equilibrium and plasma profiles, as well as on the diagnostic parameters (e.g., on the APD bias voltage and radial viewing location) and so have to be calculated explicitly for each simulated equilibrium and diagnostic setting. The validity of this procedure of utilising 2D PSFs to represent the spatial response relies on the assumption that the turbulence is to a good approximation two-dimensional, specifically that its parallel correlation length \(\ell_\parallel \gg \Delta L_b\), where \(\Delta L_b \sim 20 \text{ cm}\) is the line-of-sight path length through the beam. If the parallel correlation length \(\ell_\parallel \sim \Lambda\), where \(\Lambda \sim \pi r(B/B_\theta) \gg 1 \text{ m}\) is the connection length at the low-field side of the plasma \([29,43]\), then this approximation is well satisfied.
Figure 3: Relative density fluctuations $\left(\delta n_e/n_e\right)$ from a non-linear simulation of a MAST equilibrium (#22807 at 0.25 s) with the nominal viewing locations (●) and $1/e$ contours of the respective PSFs of the BES diagnostic superposed.

Generation of synthetic BES data from the non-linear simulations requires a 2D map of the relative density fluctuations $\delta n_e/n_e$ over a poloidal cross section (i.e. at fixed toroidal angle) as a function of time on a regular $(R, Z)$ grid covering the region of the plasma viewed by the BES diagnostic. With three viewing locations used to measure a full radial profile, the BES measurements cover a radial range of $0.95 \leq R \leq 1.45$ m, while the vertical extent of the measurements is $-0.1 \leq Z \leq +0.1$ m. Relative density fluctuations from a NEMORB simulation over such a radial stripe are shown in Fig. 3 with the PSFs of the BES system superimposed. Because the spatial extent of each channel is of order of a few cm, a resolution of $\Delta R \sim \Delta Z \sim 0.5$ cm is sufficient for the simulated data. The sampling rate of the BES diagnostic is 2 MHz, hence the sample period for the simulated data is also chosen to be $\Delta t \sim 0.5$ µs.

As well as the relative density fluctuation data, some other data is required for the synthetic data generation. Calculation of the rate of $D_\alpha$ photons incident on each sensor channel $\Gamma_{ij}(t)$ requires the mean density $n_{e0}(R, Z)$ (which does not need to be a function of time as the duration of the simulation is much shorter than the timescale of the profile evolution) and the simulated relative density fluctuation $\delta n_e/n_e(R, Z, t)$ data. The photon rate $\Gamma_{ij}(t)$ can then be calculated from the following expression:

$$\Gamma_{ij} = \int \int \varphi_{ij}(R, Z) n_{e0}(R, Z) \left[1 + B \left(\delta n_e(R, Z, t)/n_e\right)\right] dR dZ,$$

where the differential excitation rate $B = (\delta I/I)/(\delta n_e/n_e)$ was discussed in §3.1 and the integral is performed over the $(R, Z)$ region covered by the simulated data.

The statistical photon noise within the digitisation period is simulated as follows. The number of detected photons in a sample period $\Delta t = 1/f_{BW}$, where $f_{BW} \sim 5 \times 10^5$ Hz is the bandwidth of the pre-amplifiers, is $N_{ij} = Q_{eff} F_T \Gamma_{ij}/f_{BW}$, where $Q_{eff} \sim 0.85$ is the quantum efficiency of the detectors and $F_T \sim 0.25$ is a factor to take account of the transmission of the optics. Pseudo-photon noise is then added according to $N^*_ij(t) = N_{ij} + R_N \cdot \sqrt{F_N N_{ij}(t)}$, where $R_N$ is a random number from a normal distribution of unit standard deviation. The coefficient $F_N$ is the excess noise factor
due to the amplification process in the APD, which is given by $F_N = G_{\text{APD}}^\kappa \sim 2$, where $\kappa \sim 0.3$ is the excess noise exponent due to the amplification processes in the APD, and $G_{\text{APD}}$ is the gain of the APD ($G_{\text{APD}} \sim 10$ at the bias voltage used for the measurements of 310 V). The output voltage is then calculated from $V_{ij}^*(t) = \mathcal{V}_{\text{amp}} G_{\text{APD}} e N_{ij}^* f_{BW} + \Re N_{ij} \sigma_V$, where $e$ is the electron charge and $\mathcal{V}_{\text{amp}} = 3.4 \times 10^6$ V/A is the trans-impedance of the amplifiers. The second term is the electronic noise of the amplifier, also simulated by adding a normally distributed random noise voltage with standard deviation $\sigma_V = 2.5$ mV. Typically, the output voltage is $\lesssim 1.2$ V, hence the SNR due to the amplifier noise alone is $\lesssim 480$ and the overall SNR including the photon noise is $\lesssim 250$.

The procedure followed to generate the turbulence characteristics for the simulated turbulence data can be summarised as follows: PSFs for each of the three BES view radii are generated for the specific equilibrium and beam parameters used in the experiment; $\delta n_e/n_e(R, Z, t)$ data is written out from the simulation over the required domain; the synthetic BES data are generated for each available PSF; finally, the synthetic data is analysed using the same correlation analysis as for the real BES data, which was described in §3.2.

4. Comparison of non-linear simulations with measurements

Results of applying the above procedure to the five non-linear NEMORB simulations of the MAST L-mode discharge #27268 discussed in §2.2.2 are presented below and the simulated turbulence characteristics are then compared with experimental measurements. Profiles of the experimental and predicted fluctuation amplitudes $\delta n_e/n_e$ and ion heat fluxes $Q_i$ are shown in Fig. 4 as a function of normalised radius $\rho_n \equiv \psi_N^{1/2}$. Because the radial extent of the BES array covers only one third of the outboard plasma radius of $a \sim 0.5$ m, the experimental data in this figure are taken from three similar discharges, with nominally the same equilibrium and kinetic profiles, but with the BES viewing position set to three different radial locations (#27272, 1.05 m, #27268, 1.2 m and #27274, 1.35 m). The correlation analysis is applied to a $\sim 5$ ms data sample at the time $\sim 0.25$ s, corresponding to the equilibrium used for the simulations (note that this is 50 ms before the time of the beam cut-off at 0.3 s).

4.1. Turbulence amplitudes and ion heat fluxes

As shown in Fig. 4(a), the experimental fluctuation level in the core is $\sim 0.4\%$, which is significantly above that due to background light emission of $\sim 0.2\%$ and increases strongly to reach $\sim 5\%$ towards the periphery. The background fluctuation level is estimated by normalising the fluctuation amplitude just after the beam cutoff $\delta I_{bg}$ to the signal level $I_B$ just before the beam cut. Hence, $\delta I_{bg}/I_B$ should be representative of the background fluctuation level during the beam phase provided this does not change significantly between the time of observation and the beam cut-off. Spectral
Figure 4: Profiles of the normalised fluctuation amplitude $\delta n_e/n_e$ from (a) experiment (#27268) during the beam phase at 0.25 s ($\bullet$) and after beam cut-off at 0.3 s ($\circ$); and (b) from the five gyro-kinetic simulation cases in Table 1, where the plot symbols are also defined. Profiles of the gyro-Bohm normalisation ion heat fluxes $Q_{i}^{exp}/Q_{i}^{GB}$: (c) from experiment as determined by TRANSP calculations (solid line), together with the normalised neo-classical value $Q_{i}^{NC}/Q_{i}^{GB}$ (dashed line) and as estimated from the measured turbulence amplitude $\tilde{Q}_{i}/Q_{i}^{GB}$ (■); and (d) $Q_{i}^{sim}/Q_{i}^{GB}$ from the gyro-kinetic simulations.

Observations show that the background emission is predominantly so-called FIDA (fast-ion $D_\alpha$ emission) from re-neutralised beam ions, which increases towards the plasma periphery where the neutral density is highest.

The experimental heat flux, determined from transport analysis [24] performed using the TRANSP code [26], is shown in Fig. 4(b). The uncertainties in the absolute levels of $Q_{i}^{exp}$ are quite large due to the difficulty in quantifying the level of anomalous fast-ion redistribution and losses arising from fast-ion driven MHD activity, which is particularly strong during the phase with both on-axis directed NBI heating beams for which this comparison is made. Outside $\rho_n \sim 0.2$, the heat flux is approximately constant at $Q_{i}^{exp} \sim 0.02 \text{MWm}^{-2}$ and the value normalised to the gyro-Bohm level...
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$Q_i^{exp}/Q_i^{GB}$, where $Q_i^{GB} = n_i T_i v_{thi} (\rho_i / R)^2$, increases by over three orders of magnitude from $\sim 10^{-2}$ in the core to $\sim 10$ in the plasma periphery. In the outer region of the plasma, $Q_i^{exp}$ is up to a factor $\sim 5$ larger than the neo-classical heat flux $Q_i^{NC}$ determined from NCLASS [31], but it is below the neo-classical level in the core, where the power balance is particularly sensitive to assumptions made about the anomalous fast-ion diffusion.

Determining the turbulent heat flux in principle requires knowledge of the density, temperature and potential fluctuations and cross-phases between them [4]. Measurements of potential fluctuations in the core plasma are challenging and rarely available [44] and those of ion-temperature fluctuations even more so [45]. In the absence of such measurements on MAST, the gyro-Bohm-normalised turbulent ion heat flux can be estimated from the simplified relation: $\tilde{Q}_i/Q_i^{GB} \sim k_y \rho_i (T_i/T_e) (\delta n_e/n_e)^2 / (\rho_i/R)^2$, where the Boltzmann relation between $\varphi$ and $\delta n_e/n_e$ is assumed and any possible reductions due to the cross-phase are ignored. As can be seen in Fig. 4(b), this simplified estimate agrees with that determined from power balance quite well in the core and periphery but underestimates $Q_i^{exp}$ in the mid-radius region. Interestingly, measurements on TFTR [45] found that in plasmas with dominant ITG turbulence, $(\delta T_i/T_i)/(\delta n_e/n_e) \sim 2$ over the outer half of the plasma radius, so an appreciable fraction of the ion heat flux might arise from enhanced ion temperature fluctuations in regions with dominant ITG turbulence.

The normalised $E \times B$ shearing rate, $\gamma_E/(v_{thi}/a)$, is shown in Fig. 5, together with maximum growth rates $\gamma_{max}$ from linear GS2 calculations (in this case from the similar equilibrium of discharge #22807 at 0.25 s) for two cases (I and II) without sheared flow

![Figure 5: Profiles of the normalised $E \times B$ shearing rate $\gamma_E/(v_{thi}/a)$ calculated by TRANSP (solid line) and the normalised linear growth rates $\gamma_{max}/(v_{thi}/a)$ calculated by GS2 for cases I (△ – with adiabatic electrons, no flow) and II (○ – kinetic electrons, no flow or collisions).](image-url)
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or collisions. While for case I (adiabatic electrons), $\gamma_E > \gamma_{\text{max}}$ over the whole profile, for case II (kinetic electrons), $\gamma_E > \gamma_{\text{max}}$ only in the region with strong flow shear inside $\rho_n \sim 0.5$, which is consistent with the turbulence being suppressed by the flow shear there.

As shown in Fig. 4, the simulation with adiabatic electrons but without sheared flow (I) results in an ion heat flux $Q_i^{\text{sim}}$ up to an order of magnitude below $Q_i^{\text{NC}}$ and a correspondingly small fluctuation amplitude. Kinetic treatment of the trapped electrons only, again for a case without flow (II), results in values of $Q_i^{\text{sim}}$ up to an order of magnitude larger than $Q_i^{\text{exp}}$ at mid-radius, while in the core and peripheral regions $Q_i$ is underpredicted. The fluctuation level $\delta n_e/n_e$ exhibits similar behaviour. The underprediction in the core is consistent with the previous linear studies [20], which showed the region where $\dot{s} \leq 0$ to be linearly stable. The effect of including equilibrium flow shear (III) in the simulations with kinetic electrons is not dramatic, with a slight decrease in $Q_i^{\text{sim}}$ compared to case II in the core where the flow shear is strongest, but an increase in the peripheral region, where $Q_i^{\text{sim}}$ approaches the experimental level – the corresponding changes in $\delta n_e/n_e$ are rather slight.

Introducing collisions (IV and V) reduces the drive from trapped electrons, hence decreasing the level of turbulence and the heat flux. The effect of this is to reduce the radial extent over which there is significant turbulence, in the case IV (without flow) to a limited region around $\rho_n \sim 0.8$, while introducing sheared flow (V) broadens and shifts the unstable region inwards (compared to case IV) to $\rho_n \sim 0.7$. In case V, with the most complete physics, the peak value of $Q_i^{\text{sim}}$ exceeds $Q_i^{\text{exp}}$ somewhat in the mid-radius region, while the fluctuation level is close to that measured. In the peripheral region, both of these cases underpredict $\delta n_e/n_e$ and $Q_i^{\text{exp}}$ (this is not surprising because in NEMORB the fluctuation level is forced to be zero at the plasma boundary).

The fact that, particularly in the cases with collisions, the simulated turbulence has significant amplitude in relatively restricted radial regions indicates that the gradient is marginally close to the non-linear critical gradient $(R/L_{T_i})_{crit}$ to excite turbulence. This was borne out by the non-linear studies with NEMORB [20] carried out for the similar equilibrium of discharge #22807 (see §2.2), which showed that relatively small increases in $R/L_{T_i}$ above the experimental value could produce large increases in the turbulent heat flux. Although such calculations have not been carried out for the equilibrium used for these studies, comparison of similar non-linear simulations for both equilibria show $R/L_{T_i}$ to be even closer to marginality in the equilibrium from #27268 due to the relatively smaller predicted heat fluxes.

Using our database of equilibrium and turbulence data, primarily from MAST L-mode discharges, it is found [28, 30] that $R/L_{T_i}$ exhibits a dependence on flow shear and magnetic geometry, specifically on the parameters $q/\epsilon$ and $\tilde{\gamma}_E = \gamma_E/(v_{\text{thi}}/R)$. This dependence shows remarkable similarity to a numerically predicted ‘manifold’ of $R/L_{T_i}(q/\epsilon, \tilde{\gamma}_E)$, required for the excitation of marginally unstable ITG- or PVG-driven turbulence [46]. This observation is further evidence that the $R/L_{T_i}$ is generally close to marginal stability in MAST L-mode plasmas. A consequence of this closeness of
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Figure 6: Profiles of (a) perpendicular $\ell_y$ and (b) radial $\ell_x$ correlation lengths determined from experiment ($\bullet$) and from the simulations for the five cases, where the symbols are defined in Table 1.

$R/L_T$ to marginality is that the use of the heating operator in NEMORB to match the predicted $T_i$ profile to that prescribed will yield heat fluxes which are very sensitive to experimental uncertainties. Hence, it is not surprising that there is not better agreement between predicted profile of $Q_i^{\text{sim}}$ and experiment.

The shortfall in the predicted level of turbulence and heat flux $Q_i^{\text{sim}}$ in the peripheral region is a clear deficiency of these non-linear NEMORB simulations, which probably arises from the necessary boundary conditions (see §2.2.3) that suppress any turbulence at the plasma periphery. As mentioned in the introduction, such a shortfall has also been found in local, non-linear simulations of DIII-D L-mode discharges using the global code GYRO [2, 12, 13]. In these studies, although good agreement was found in the plasma core between simulated and experimental ion and electron heat fluxes and turbulence characteristics, a systematic shortfall in the heat flux and fluctuation levels was observed at $r/a \sim 0.8$ [12]. More recently, concerted efforts have been made by several groups to either understand or resolve this discrepancy [47, 48] and currently this apparent shortfall appears not to be ubiquitous.

4.2. Perpendicular and radial correlation lengths

As shown in Fig. 6, the measured perpendicular correlation length $\ell_y \sim 10 - 20$ cm is approximately constant with radius, whereas the radial correlation length $\ell_x \sim 2 - 7$ cm exhibits a minimum at $\rho_n \sim 0.7$, increasing monotonically further outwards and exhibiting step-like increase to a constant value inside this radius. The observed degree of anisotropy $\ell_y/\ell_x \sim 3 - 4$ is consistent with the findings of Ref. [29]. The corresponding values of $k_{x,y}\rho_i$ (where $k_{x,y} = 2\pi/\ell_{x,y}$) are $k_y\rho_i \sim 0.3 - 1.0$ and $k_x\rho_i \sim 0.5 - 2$, generally decreasing with increasing radius.

In all of the simulated cases, the perpendicular correlation length $\ell_y$ is comparable
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to that observed over most of the profile except in the periphery \((\rho_n > 0.8)\), where the observed \(\ell_y\) are shorter and the global simulations may be impacted by the boundary conditions. Comparison of radial correlation lengths \(\ell_x\) is complicated by the non-monotonic form of the observed profile. The simulated \(\ell_x\) are mainly shorter than those observed, except in the periphery \((\rho_n > 0.8)\) where the observed \(\ell_x\) increases with radius while the predicted \(\ell_x\) decreases. The simulated turbulence hence exhibits a greater degree of anisotropy than that observed experimentally.

4.3. Correlation times

The measured correlation times, shown in Fig. 7, are very short and in the range \(\tau_c \sim 1 - 6 \mu s\). One of the most striking observations is that in all of the simulations, the predicted correlation times \(\tau_c\) are almost two orders of magnitude longer than those measured. Note that the values of \(\tau_c\) for the simulation cases in Fig. 7 had to be calculated using synthetic data generated without adding noise. This is because, when the turbulence amplitude is weak, the determination of \(\tau_c\) is particularly subject to the effect of noise on the correlation functions and not many valid data points remained for cases IV and V.

It was shown in Ref. [29] that the measured \(\tau_c\) are comparable with various linear time scales. These time scales are: the drift-wave time \(\tau_{st}^{-1} \sim \omega_{si}\), the parallel streaming time \(\tau_{st}^{-1} \sim v_{thi}/\Lambda\) (thermal ion transit time over the parallel connection length \(\Lambda\)) and the magnetic drift time \(\tau_{M}^{-1} = v_{thi}\rho_i/(\ell_xR)\) (perpendicular magnetic drift time over a radial correlation length \(\ell_x\)). This observation is consistent with the turbulence being ‘critically balanced’ [43] and implies that the turbulence is anisotropic in the poloidal plane with \(\ell_y/\ell_x \sim R/L_*\), where \(L_*\) is taken as the shorter of \(L_{Ti}\) and \(L_{ne}\) [29]. It can be seen from Fig. 6 that such an anisotropy is observed experimentally and to an even greater degree in the NEMORB simulations. It was also found that the observed

Figure 7: Profiles of the correlation time \(\tau_c\) determined from experiment (●) and from the NEMORB simulations for the five cases, where the symbols are defined in 1.
correlation times $\tau_c$ are always shorter than or comparable to the $E \times B$ shearing time $\tau_{sh} \sim 1/\gamma E$, i.e., $\tau_c \leq \tau_{sh}$, which implies that equilibrium $E \times B$ shear does not always determine $\tau_c$ in the experiment.

In Fig. 8, the correlation times $\tau_c$ for the L-mode experimental data considered here, together with those from the five NEMORB simulations, are compared with these time scales. For this experimental data, although $\tau_s \sim \tau_{st} \sim \tau_M \sim \tau_{sh}$, the observed $\tau_c$ are usually somewhat shorter than the various linear time scales, which is consistent with strong, critically balanced turbulence. In the case of the simulation results, the derived $\tau_c$ are substantially longer than any of the linear time scales. This indicates that in these simulations, the turbulence is weak (whereby $\tau_c/\omega_{ce} \gg 1$), in contrast to the strong turbulence measured in the experiment.

The de-correlation of the turbulence by the fluctuating potential from the drift waves $\varphi^{\text{DW}}$ is characterised by the non-linear time $\tau_{nl}^{-1} \sim v_{thi} \rho_i \varphi^{\text{DW}} / (\ell_x \ell_y)$, which can be estimated from the turbulence characteristics by assuming $\delta n_e / n_e$ and $\varphi^{\text{DW}}$ are related through the Boltzmann response. Under this assumption, $\varphi^{\text{DW}}$ does not include any contribution from the trapped-electron response or toroidally and poloidally symmetric zonal flows, so this timescale is denoted $\tau_{nl}^{\text{NZ}}$ and given by $(\tau_{nl}^{\text{NZ}})^{-1} \sim v_{thi} \rho_i / (\ell_x \ell_y) \cdot (T_e/T_i) \delta n_e / n_e$. In Ref. [29], it was found that the measured correlation times $\tau_c$ were generally much shorter than $\tau_{nl}^{\text{NZ}}$ and it was argued that this could be consistent with a strong zonal component to the turbulence $\varphi^{\text{ZF}}$ contributing dominantly to its decorrelation (the ratio $\tau_c / \tau_{nl}^{\text{NZ}}$ was also found to increase with collisionality, suggesting that the ratio of the zonal to the drift-wave component of the turbulent amplitude $\varphi^{\text{ZF}} / \varphi^{\text{DW}}$ increases with decreasing collisionality). In the experiment considered here, we also find that the non-linear time $\tau_{nl}^{\text{NZ}}$ are always substantially longer than $\tau_c$ (see Fig. 8(e)). In contrast, in our simulations, the non-linear times $\tau_{nl}^{\text{NZ}}$ are mostly comparable to $\tau_c$, which indicates that the simulated turbulence is decorrelated by the drift-wave potential fluctuations $\varphi^{\text{DW}}$ and not by a dominant
Figure 9: Profiles of the poloidal propagation velocity $U_Z^{BES}$ of the fluctuating density patterns for (a) cases without equilibrium flow, where the local $E \times B$ velocity is assumed to be $U_Z^{E \times B} = 0$ and (b) for cases with equilibrium flow, including from experiment ($\bullet$), where the symbols for simulated cases are defined in Table 1. The local $E \times B$ velocity $U_Z^{E \times B} \approx -U_\phi \tan \alpha$ is shown in (b) as the dot-dashed line and the electron- and ion diamagnetic velocities with respect to $U_Z^{E \times B}$, i.e., $U_Z^{E \times B} + U_Z^{dia,i}$ by the dotted and dashed lines respectively.

component from zonal flows.

4.4. Poloidal propagation velocities

The apparent poloidal velocity of the density perturbations is shown in Fig. 9(b) together with the poloidal component of the local $E \times B$ velocity, $U_Z^{E \times B} \approx -U_\phi \tan \alpha$, where $U_\phi$ is the toroidal flow velocity measured using CXRS. The poloidal electron and ion diamagnetic velocities $U_Z^{dia,e,i} \sim T_{e,i} / (L_{P_{e,i}} B) / \cos \alpha$, where $L_{P_{e,i}}$ are the electron- and ion pressure-gradient-scale lengths are estimated using the TS and CXRS profile measurements assuming a pure plasma ($n_i = n_e$). In Fig. 9, we plot the sum $U_Z^{E \times B} + U_Z^{dia,i}$ and compare it with the apparent poloidal velocity of the density patterns as measured by BES (see §3.2). It can be seen that the observed density patterns propagate in the ion-diamagnetic direction relative to $U_Z^{E \times B}$ at a poloidal velocity $U_Z \sim U_Z^{BES} - U_Z^{E \times B}$ of the order of the ion diamagnetic velocity, $U_Z^{dia,i}$. The observation of radially sheared poloidal flow at mid-radius is perhaps evidence for the presence of zonal flows in this regime.

Poloidal velocity profiles $U_Z^{BES}$ from the simulations without equilibrium flow are shown in Fig. 9(a), where $U_Z^{E \times B} = 0$. In case I (adiabatic electrons), the drift velocity is small. In simulations II and III (kinetic electrons, no collisions), the density fluctuations propagate at approximately the electron-diamagnetic drift velocity $U_Z^{dia,e}$ with respect to $U_Z^{E \times B}$, which is consistent with the turbulence being driven by the trapped electrons, and in the opposite direction to that observed. In the cases with collisions, the trapped
Comparison of turbulence measurements with direct gyrokinetic simulations

5. Conclusions

This validation exercise, comparing synthetic BES data generated from global, non-linear gyro-kinetic simulations with observations, has revealed that the simulations with the most complete physics, i.e., including kinetic-electrons, sheared equilibrium flow and collisions, exhibit a degree of agreement in terms of the radial and poloidal correlation lengths, the ion heat flux and fluctuation levels (at mid-radius) and the poloidal propagation velocity of the turbulence. There are, however, notable discrepancies that suggest that the simulations have a long way to go before they can be viewed as reliably reproducing reality. Firstly, the measured correlation times are very short, consistent with strong turbulence and the approximate equality between these correlation times and the linear time scales (drift-wave drive, parallel streaming, magnetic drift), indicates that the turbulence is critically balanced, whereas the simulated turbulence exhibits much longer correlation times and is only weakly non-linear. Secondly, the predicted profiles of fluctuation level and turbulent heat flux exhibit some degree of agreement with experiment only in the mid-radius region. This is partly because neo-classical physics is not included in the NEMORB simulations but also because, under conditions where the gradients are close to marginal stability, the prescribed ion-temperature gradient would have to match very precisely in order to yield the experimental heat flux. Finally, the simulations exhibit a marked shortfall in the predicted fluctuation level and ion heat flux in the peripheral region, which is likely both to be a result of the ‘zero-turbulence’ boundary condition required to ensure numerical stability and quite plausibly some missing physics related to non-local interaction of peripheral turbulence with edge/SOL instabilities. Further work is planned to understand these result and to improve the simulations, e.g. to investigate the sensitivity of the predicted heat flux to local changes in the ion-temperature gradient, to resolve the components of the heat flux due to density and ion temperature fluctuations and to examine the possible role of parallel dynamics.

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