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A gyrokinetic calculation of transmission and reflection of the fast wave in the ion cyclotron range of frequencies

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A full-wave equation has been obtained from the gyrokinetic theory for the fast wave traversing a minority cyclotron resonance [Phys. Fluids B 4, 493 (1992)] with the aid of the fast wave approximation [Phys. Fluids 31, 1614 (1988)]. This theory describes the transmission, reflection, and absorption of the fast wave for arbitrary values of the parallel wave number. For oblique propagation the absorption is due to both ion cyclotron damping by minority ions and mode conversion to the ion Bernstein wave. The results for a 3He minority in a D plasma indicate that for perpendicular propagation and minority temperatures of a few keV the power lost by the fast wave is all mode converted whereas for minority temperatures ~100 keV ~30% of the incident power is dissipated by the minority ions due to the gyrokinetic correction. The gyrokinetic correction also results in a significant reduction in the reflection coefficient for low field side incidence when \( k_{\parallel}L_B \ll 1 \) and the minority and hybrid resonances overlap.

I. INTRODUCTION

As magnetic fusion approaches its goal of an ignited plasma, energetic ions whose Larmor radii are comparable to the wavelength of the fast magnetoacoustic wave will necessarily be present. The spatial width of an ion cyclotron resonance is of order \( k_{\parallel}L_B \rho_i \) where \( L_B \) is the scale length of the equilibrium magnetic field, \( k_{\parallel} \) is the wave number parallel to this field, and \( \rho_i \) the ion Larmor radius. For \( k_{\parallel}L_B \gg 1 \), the resonance width is many Larmor radii. However, for \( k_{\parallel}L_B \ll 1 \), the resonance is much narrower, and it is of interest to consider the variation of the equilibrium magnetic field across an ion Larmor radius. The inclusion of this variation is in any case required, in order to construct the self-consistent particle response to an electromagnetic perturbation in a nonuniform magnetic field.4,5 In addition, the inclusion of the variation of the magnetic field across an ion Larmor orbit has recently been shown6 to lead to an unambiguous and concise technique for deriving energy-conserving, full-wave equations. A number of other authors6,7,9 have also solved for the self-consistent kinetic response in a nonuniform field, but different orderings were chosen and it is not clear whether the variation across the Larmor radius was included. In Refs. 2 and 3, Lashmore-Davies and Dendy applied the gyrokinetic theory of Chen and Tsai8 to obtain the self-consistent local dispersion relation for the fast wave propagating perpendicular to a nonuniform magnetic field. This approach has now been generalized10 to oblique propagation, and the local theory has been used to calculate the transmission coefficient of the fast wave across an ion cyclotron resonance. However, such a procedure is not capable of yielding information on the reflected energy.

In this paper we shall employ the method developed in Refs. 11 and 12 to construct a full-wave equation for the fast wave, using the self-consistent gyrokinetic dispersion relation. This equation yields the transmission and reflection coefficients predicted by gyrokinetic theory, where the resulting “power absorbed” is the sum of the power mode converted and the power truly dissipated by the resonant particles. By comparing these results with those of a locally uniform analysis, where for almost perpendicular incidence the “power absorbed” is due solely to mode conversion, it is possible to infer the magnitude of the true dissipation.

The plan of the paper is as follows. In Sec. II, we use the results obtained from the gyrokinetic analysis10 to obtain the fast wave equation. In Sec. III, we discuss the conservation property of the fast wave equation, and in Sec. IV we present a representative sample of results obtained from a numerical solution of this equation. This is followed by a discussion of these results and our conclusions.

II. THE GYROKINETIC FAST WAVE EQUATION

Let us first derive the gyrokinetic fast wave equation, using the results given in Ref. 10 where further details may be found. The nonuniform magnetic field that we consider10 is the simplest one relevant to a tokamak:

\[
B = e_z B_0 (1 + x/L_B).
\]

\[ (1) \]
Here \( z \) may be interpreted as the toroidal direction, \( x \) as the radial, and \( y \) as the poloidal. We simplify the discussion by restricting the analysis to wave vectors \( \mathbf{k} = (k_x,0,k_y) \). We also make use of the usual approximation for the fast wave, and neglect the parallel component of the electric field \( \delta E_z \).

The perturbed perpendicular currents may then be written

\[
\delta J_{sk} = -\sigma_{xx}\delta E_{sk} + \sigma_{xy}\delta E_{yk},
\]

\[
\delta J_{yk} = \sigma_{yx}\delta E_{sk} + \sigma_{yy}\delta E_{yk},
\]

and the elements of the conductivity tensor may be written

\[
\sigma_{ij} = (\sigma_{ij})_{NR} + (\sigma_{ij})_{R},
\]

where \( R \) refers to the resonant ion species in a plasma with two species of ions, and \( NR \) refers to all nonresonant particles. We restrict discussion to the case where the second ion species is not resonant at the same location in the non-uniform magnetic field, for example, \( D(\text{He}) \). The nonresonant conductivity elements are standard\(^{10}\) and may be written

\[
(\sigma_{xx})_{NR} = \frac{i\omega_{pe}^2}{4\pi}\frac{\sigma_0}{\Omega_0}\left(\frac{r_1 - r_2}{(r_1^2 - 1) + \frac{r_2}{4}}\right),
\]

\[
(\sigma_{xy})_{NR} = -\frac{i\omega_{pe}^2}{4\pi}\frac{\sigma_0}{\Omega_0}\left(\frac{r_1^2}{(r_1^2 - 1) + \frac{r_2}{4}}\right),
\]

\[
(\sigma_{yx})_{NR} = - (\sigma_{xy})_{NR},
\]

\[
(\sigma_{yy})_{NR} = (\sigma_{xx})_{NR}.
\]

Here \( r_1 = \Omega_1/\Omega_0, r_2 = n_{a0}Z_a/n_{0a}Z_a \) and the nonresonant (majority) ions are denoted by subscript \( "a" \) and the resonant (minority) ions by subscript \( "b" \). Equations (5)-(8) contain the contributions from the electrons, the majority, and those minority ions that are not resonant. The resonant conductivity elements are given by\(^{10}\)

\[
(\sigma_{xx})_R = iQK_1,
\]

\[
(\sigma_{xy})_R = -QK_2,
\]

\[
(\sigma_{yy})_R = iQK_3,
\]

where

\[
Q = -\frac{\tau}{k_s^2} \frac{\omega_{pe}^2}{8\pi} \exp\left(-\frac{k_s^2 \rho_b^2}{4}\right),
\]

\[
K_1 = Z(\eta_{1b}).
\]

\[
K_2 = \left(1 - \frac{k_s^2 \rho_b^2}{2\tau}\right)Z(\eta_{1b}) + 2\nu\left(\frac{i\omega}{\tau} + \nu\right)\left[1 + \eta_{1b} Z(\eta_{1b})\right],
\]

\[
\eta_{1b} = \tau\left(\frac{\omega - \Omega_b(x)}{k_s^2} - \frac{ik_s \rho_b}{2\nu}\right),
\]

\[
\nu = 1/k_s L_B.
\]

Substituting Eqs. (2) and (3) into Maxwell's equations, we obtain

\[
\left(\nu^2 + 4\pi\omega_e/c^2\right)\delta E_{sk} - \frac{4\pi\omega_e}{c^2}\eta_{xx}\delta E_{yk} = 0,
\]

\[
\left(\nu^2 + 4\pi\omega_e/c^2\right)\delta E_{yk} + \left(\frac{\omega^2}{c^2} - \frac{4\pi\omega_e}{c^2}\sigma_{yy}\right)\delta E_{sk} = 0.
\]

Eliminating \( \delta E_{sk} \) in favor of \( \delta E_{yk} \), we obtain the equation

\[
k_s^2 \delta E_{yk} + \frac{\left\{\left[k_s^2 - (\omega^2/c^2) - (4\pi\omega_e/c^2)\sigma_{xx}\right]\left[k_s^2 - (\omega^2/c^2) - (4\pi\omega_e/c^2)\sigma_{yy}\right] + (4\pi\omega_e/c^2)^2\sigma_{xy}\sigma_{yx}\right\}}{\left[k_s^2 - (\omega^2/c^2) - (4\pi\omega_e/c^2)\sigma_{xx}\right]} \delta E_{yk} = 0.
\]

Equation (21) gives the fast wave refractive index. It is also the self-consistent local dispersion relation obtained from gyrokinetic theory to order \( k_s^2 \rho_b^2 \). We may convert Eq. (21) into a second-order differential equation\(^{11,12}\) for the fast wave electric field amplitude \( \delta E_k \) by substituting the "cold" solution \( k_s \) in the resonant conductivity tensor elements, and replacing \( k_s \delta E_{yk} \) by \((-d^2/dx^2)\delta E_y\). Thus, the required second-order fast wave equation can be written

\[
d^2/dx^2 \delta E_y + V(x)\delta E_y = 0.
\]

With the aid of Eqs. (5)-(12), the fast wave potential \( V(x) \), which is defined by Eqs. (21) and (22), can be put in the form

\[
V(x) = -\omega^2 \left[\frac{Q_a^2}{2}\left(N^2 - \frac{c^2}{c_f^2}\right) + 2\left(N^2 - \frac{c^2}{c_f^2}\right)\frac{\Omega_a}{\omega}\left(\frac{r_1}{(r_1 + 1) + \frac{r_2}{4}}\right) - \frac{\Omega_a^2}{\omega^2}\left(\frac{r_1}{(r_1 + 1) + \frac{r_2}{4}}\right) + \frac{\tau\Omega_a^2}{2\omega k_s \nu \tau_b} \right]
\]

\[
\times \left[\frac{Q_a}{\omega}\left(\frac{r_1}{(r_1 + 1) + \frac{r_2}{4}}\right) - \left(N^2 - \frac{c^2}{c_f^2}\right) r_1 r_2 (K_1 + \tau^2 K_2)\right]^{-1}.
\]
where
\[ N_s = e_i k_x/\omega. \]  
(24)

This fast wave potential is used in our numerical solutions of Eq. (22), and is valid for arbitrary minority to majority density ratios. Solutions of Eq. (22), with appropriate boundary conditions, will give the transmission, reflection, and absorption coefficients for the fast wave incident from either side of the minority resonance. Since two ion species are present, the two ion hybrid resonance can also occur.

III. THE FAST WAVE CONSERVATION RELATION

It is straightforward to obtain from Eq. (22) the conservation relation
\[ \frac{d}{dx} \left[ \text{Im}(\delta E_y) \right] = -|\delta E_y|^2 \text{Im}(V(x)). \]  
(25)

The right-hand side of Eq. (25) gives the power lost by the fast wave in traversing the interaction region. As discussed in Ref. 11, the power lost is the sum of the power dissipated by the minority ions due to the fundamental minority wave particle resonance and the power lost due to a wave (two-ion hybrid) resonance. The new feature of the gyrokinetic theory is an additional dissipation mechanism, which exists even for \( k_B L_B = 0 \). It is due to the variation of the equilibrium magnetic field across the Larmor orbits of the minority ions. This has the interesting consequence that the pole in the potential \( V(x) \), which corresponds to the hybrid resonance, no longer occurs on the real \( x \) axis even for perpendicular incidence. As a result, \( \text{Im} V(x) \) may be integrated through the hybrid resonance without the need to deform the contour along the real \( x \) axis, as is necessary in the locally uniform theory.\(^{11}\) This property of the gyrokinetic fast wave potential is clear, since even in the asymptotic case of large argument of the \( Z \) function that appears in the potential \( V(x) \) the functions \( K_1 \) and \( K_2 \) defined by Eqs. (14) and (15) retain an imaginary part directly proportional to the imaginary part of \( \eta_{ib} \), namely \( k_x \rho_B^2 \eta_{ib}/2 \). Hence the sign of \( \text{Im} V(x) \) depends on the choice of the sign of \( k_x \) in \( \eta_{ib} \).

It is worth considering this point explicitly. We therefore obtain the asymptotic form of the potential for small minority ratios by expanding the \( Z \) function, using the fact that
\[ \frac{[\omega - \Omega_b(x)]}{k_x \rho_B^2} > k_x \rho_B^2/2 \]
far from the minority resonance. This is equivalent to the condition \( |x|/\rho_B > k_x \rho_B^2/2 \). Having carried out this expansion, the asymptotic forms of the \( Z \) function and its derivative lead to
\[ V(x) \approx \frac{\omega_t^2}{\rho_B^2} \frac{1}{(r_1 - 1)} \frac{r_2}{2r_1 (r_1 + 1) N_e v_T} \frac{r}{2} c_A \]
\[ \times \left[ \rho_B^2 \left( 1 + \tau^2 + \nu^2 + \frac{k_x^2 \rho_B^2}{2} + \frac{k_x^2 \rho_B^2}{2} \right) \right] \]
\[ + \frac{ik_x \rho_B \nu \tau q_{ib}^2}{2x^2} \left( 3 + \tau^2 + \nu^2 \tau^2 \right) \left[ \frac{1}{(r_1 - 1)} \right] \]
\[ + \frac{r_2 r}{2} \frac{c_A}{N_e v_T} \left( -\frac{\rho_B^2}{x} + \frac{ik_x \rho_B \nu \tau q_{ib}^2}{2x^2} \right) \right]^{-1} \]
(26)

where
\[ q = (k_x^2 L_B^2 + 1)^{1/2}. \]  
(27)

The hybrid resonance occurs at \( x = x_R \) where, from the denominator of Eq. (26),
\[ \frac{1}{(r_1 - 1)} - \frac{r_2 q}{2r_1 N_e v_T x} = 0. \]  
(28)

It then follows from Eq. (26) that
\[ \text{Im}\{V(x_R)\} = 2 \frac{\omega_t^2}{\rho_B^2} (r_1 - 1) x_R \]
\[ - \frac{r_2}{2} \frac{c_A}{k_x \rho_B} \frac{1}{(r_1 + 1)} \frac{1}{\rho_B}. \]  
(29)

For both \( k_B L_B < 1 \) and \( k_B L_B > 1 \), Eq. (29) reduces to
\[ \text{Im}\{V(x_R)\} = 2 \frac{\omega_t^2}{\rho_B^2} (r_1 - 1) x_R \]
\[ - \frac{r_2}{2} \frac{c_A}{k_x \rho_B} \frac{1}{(r_1 + 1)} \frac{1}{\rho_B}. \]  
(30)

here a term in \( k_x^2 \rho_B^2 \) has been neglected in comparison with unity at the upper limit. We have noted from Eq. (25) that the power absorbed depends on the sign of \( \text{Im} V(x) \). Equation (30) appears to show that the sign of \( \text{Im} V(x_R) \) depends on the quantity \( (r_1 - 1) \), which is positive for a “light” minority (higher cyclotron frequency). However, this is not so: by Eq. (28), the resonance position also changes sign, so that the sign of \( \text{Im} V(x_R) \) is determined solely by the choice of cold solution \( k_x = \pm k_x^c \) where \( k_x^c > 0 \). In order to ensure loss of power as the wave crosses the resonance, we require \( \text{Im} V(x_R) > 0 \) and hence must choose \( k_x = + k_x^c \). Using Eq. (28), we may then write
\[ \text{Im}\{V(x_R)\} = 2 \frac{\omega_t^2}{\rho_B^2} (r_1 - 1)^2 \frac{r_2}{2} \frac{L_B}{\rho_B}. \]  
(31)

The high field side is defined by Eq. (1) to be \( x > 0 \). Since we have previously assumed a harmonic variation \( \exp (i(k_x x - \omega t)) \), where \( \omega > 0 \), \( \exp(ik_x x) \) represents a wave traveling from \( -x \) to \( x \). Thus, for a wave incident from the high field side, integration of Eq. (25) yields
\[ 1 = T + \frac{1}{k_x} \int_{-\infty}^{\infty} \text{Im} V(x) |\delta E_y(x)|^2 dx; \]  
(32)

whereas for a wave incident from the low field side, Eq. (25) gives
\[ 1 = T + R + \frac{1}{k_x} \int_{-\infty}^{\infty} \text{Im} V(x) |\delta E_y(x)|^2 dx. \]  
(33)

where \( T \) and \( R \) are the transmission and reflection coefficients. Equations (32) and (33) show that we have made the correct choice in Eq. (31) to obtain wave damping.
IV. RESULTS AND DISCUSSION

In this section, we present results obtained from numerical solution of Eq. (22). First, let us consider the case shown in Figs. 1 and 2. These refer to a D(3He) plasma for Joint European Torus (JET) parameters ($B_0=3.4$ T, $L_s=3.1$ m) where the $^3$He temperature is 1.77 keV and the minority ion to electron density ratio is 0.05. Figure 1 shows the transmission coefficients as a function of $k_z$ predicted by the fast wave equation, both when the fast wave potential is obtained from locally uniform theory, and when it is obtained from gyrokinetic theory as described in this paper. It can be seen that for this minority temperature the two theories are in agreement over the whole range of $k_z$.

Figure 2 shows the reflection coefficients as a function of $k_z$ and again the two theories are in agreement over the whole range.

It was noted in Ref. 10 that the perpendicular damping mechanism introduced by gyrokinetic theory becomes stronger as the minority temperature increases. This is evidently due to the fact that the minority resonance is broadened with increasing minority temperature, so that it can overlap the hybrid resonance for larger values of $n_0/n_e$ and hence smear out the effects of the hybrid resonance. It was conjectured$^{10}$ that as the minority temperature continues to increase, the optical depth of the minority resonance should depend only on the parameter $v_T/v_A$. We have therefore solved Eq. (22) for a minority temperature of 100 keV for a range of values of $n_0/n_e$ where $n_e$ is the equilibrium electron density and compared the transmission, reflection, and power absorption coefficients as functions of $k_z$ with those obtained from the locally uniform theory. The results for $n_0/n_e=0.05$ are shown in Figs. 3 and 4 where the calculations refer to low field-side incidence, and the other parameters are the same as in Figs. 1 and 2. Figure 3 shows the dependence of the transmission, reflection, and absorption coefficients on $k_z$ obtained from the locally uniform model and Fig. 4 shows the corresponding results obtained from the gyrokinetic model. It is clear that there are now significant differences.

FIG. 1. Transmission coefficients from locally uniform and gyrokinetic full-wave models vs $k_z$ for D($^3$He). Plasma parameters $B_0=3.4$ T, $L_s=3.1$ m, $T_{^3He}=1.77$ keV, $n_0/n_e=0.05$, $n_0=5\times10^{19}$ m$^{-3}$, and $f=34.5$ MHz.

FIG. 2. Reflection coefficients for locally uniform and gyrokinetic full-wave models vs $k_z$ for D($^3$He). Parameters as for Fig. 1.

FIG. 3. Transmission, reflection, and absorption coefficients for locally uniform full-wave model vs $k_z$ for D($^3$He) and power incident from the low field side. Plasma parameters are $B_0=3.4$ T, $L_s=3.1$ m, $T_{^3He}=100$ keV, $n_0/n_e=0.05$, $n_0=5\times10^{19}$ m$^{-3}$, and $f=34.5$ MHz.

FIG. 4. Transmission, reflection, and absorption coefficients for gyrokinetic full-wave model vs $k_z$ for D($^3$He) with the power incident from the low field side. Parameters as for Fig. 3.
between the two theories, particularly for small values of \(k_z\). The gyrokinetic theory predicts lower reflection and higher absorption than the locally uniform model although these differences become progressively less pronounced for \(k_z > 5 \text{ m}^{-1}\). An additional feature illustrated by Figs. 3 and 4 is the role of the perpendicular minority ion dissipation. In the locally uniform theory there is no perpendicular dissipation—the "power absorbed" is due to mode conversion, that is, energy lost at the two-ion hybrid resonance. This is borne out by comparing the reflection and absorption coefficients (for \(k_z = 0.1 \text{ m}^{-1}\)) with \((1 - T)^2\) and \(T(1 - T)\), respectively, which reproduce the calculated values. On the other hand, for the gyrokinetic coefficients, the reflection is less than \((1 - T)^2\) and the power absorbed is larger than \(T(1 - T)\). The comparison is given in Tables I and II for the local and gyrokinetic theories, respectively, which suggests that the perpendicular ion dissipation is responsible for these differences. This leads to the interpretation that the ion dissipation is 2%, 17%, 23%, 20%, and 4% for the minority ratios 0.01, 0.03, 0.04, 0.05, and 0.06, respectively. Table III provides further evidence for this interpretation where the gyrokinetic reflection and absorption coefficients are again compared with \((1 - T)^2\) and \(T(1 - T)\) for perpendicular propagation (\(k_z = 0.1 \text{ m}^{-1}\)) and a minority ratio of 0.04. The results for four minority temperatures, 5, 50, 100, and 200 keV, are given.

The results for 5 keV are almost identical to the locally uniform calculation. However, for the higher temperatures the reflection decreases and the absorption increases as the minority temperature increases. This behavior can again be interpreted as being due to the perpendicular ion dissipation mechanism. Note that the transmission coefficient falls with increasing minority temperature in contrast to the locally uniform model where the transmission coefficient is independent of temperature. For a minority temperature of 200 keV the perpendicular ion dissipation reaches a value of 38% for the minority ratio 0.04. It is also worth pointing out that the reduction in the reflection coefficient predicted by gyrokinetic theory only occurs when the minority cyclotron resonance overlaps the hybrid resonance. When the resonances are well separated, the gyrokinetic theory predicts a larger reflection coefficient than the locally uniform model. This is demonstrated by the case \(n_{0b} / n_{0c} = 0.06\) in Tables I and II. Figures 5 and 6 show the dependence of the transmission, reflection, and absorption coefficients on \(k_z\) over the range of \(k_z\) where the difference between the local theory (Fig. 5) and gyrokinetic theory (Fig. 6) is most noticeable.

In conclusion, we have found that the gyrokinetic theory produces significant differences from the locally uniform theory, particularly for \(k_z L_s \lesssim 1\) and for high minority ion temperatures. Gyrokinetic theory reduces the transmission coefficient, which is the same for either high or low field-side incidence. It also reduces the reflection coefficient from the low field side provided the minority cyclotron resonance overlaps the hybrid resonance. The most striking change introduced by the gyrokinetic theory is an additional dissipation mechanism, which is present at all angles of propagation including perpendicular where its effects are most noticeable. By comparing the reflection

<table>
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<tr>
<th>(n_{0b} / n_{0c})</th>
<th>(T)</th>
<th>(R)</th>
<th>((1 - T)^2)</th>
<th>(A)</th>
<th>(T(1 - T))</th>
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<td>0.211</td>
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<th>(T) (keV)</th>
<th>(T)</th>
<th>((1 - T)^2)</th>
<th>(A)</th>
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<td>0.540</td>
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FIG. 5. Transmission, reflection, and absorption coefficients for locally uniform full-wave model vs \(k_z\) over restricted range of \(k_z\) for D(He) and power incident from the low field side. Plasma parameters as for Fig. 3.
that would be produced in the absence of this perpendicular ion dissipation with that actually found, the power dissipated by this process is inferred. However, for high field-side incidence, we have not been able to find a simple method to separate the power dissipated from that mode converted at the hybrid resonance. In order to do this we must solve the corresponding fourth-order differential equation that we have already derived from the gyrokinetic analysis.

In view of the significant reduction in the amplitude of the incident fast wave in crossing the minority resonance, it is worth noting that the fast wave differential equation, Eqs. (22) and (23), has been derived from gyrokinetic theory assuming $k_x \rho_b \ll 1$. This condition is well satisfied outside the resonance region, but may well be violated inside this region. The present analysis suggests that changes over a few ion Larmor radii produce important qualitative effects, particularly for high minority ion temperatures. In order to quantify these effects fully a theory valid for arbitrary values of $k_x \rho_b$ is required. This gives rise to an integrodifferential equation. We have generalized our analysis to this important case and work is now in progress on the solution of the arbitrary $k_x \rho_b$ problem.

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