

The Effects of Sheared Toroidal Rotation on Stability Limits in Tokamak Plasmas

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Sheared toroidal rotation is found to increase the ideal external kink stability limit, thought to be the ultimate performance limit in fusion tokamaks. However, at rotation speeds approaching a significant fraction of the Alfvén speed, the toroidal rotation shear drives a Kelvin-Helmholtz-like global plasma instability. Optimising the rotation profile to maximise the pressure before encountering external kink modes, but simultaneously avoiding flow-driven instabilities, can lead to a window of stability attractive for operating future high-performance fusion devices.

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A future magnetically confined fusion power plant is predicated upon achieving non-inductively sustained plasmas operating at high β_N [1, 2], where $\beta_N = 2\mu_0\langle p\rangle a/B_0 I_p$ [MA], $\langle p\rangle$ is the volume averaged pressure, B_0 is the toroidal magnetic field, a is the minor radius and I_p is the plasma current. Previous power plant optimisation studies [3] concluded that the cost of electricity scales with $\beta_N^{-0.4}$, making high β_N operation attractive. Furthermore, high temperature, and consequently high β_N , plasmas are also desirable to enhance current drive efficiency and maximise the plasma-generated non-inductive bootstrap current [4].

Modern-day tokamaks heated by neutral beam injection (NBI) can rotate toroidally at a significant fraction of the Alfvén speed [5]. Such strong toroidal rotation can ameliorate or suppress many magnetohydrodynamic (MHD) instabilities, including resistive wall modes [6, 7], edge localised modes [8, 9] and the internal kink mode [10, 11]. Spherical tokamaks (STs) can have toroidal flows approaching the ion sound speed due to the small plasma volume per unit of NBI power and low moment of inertia of the plasma. Future STs, such as the Component Test Facility (ST-CTF) [12, 13] are likely to rotate extremely quickly. At such large sheared rotation velocities, magnetic Kelvin-Helmholtz instabilities [14] become a concern, as recently observed in a magnetised plasma in the solar corona [15]. In the same way that previous studies have investigated shaping or aspect ratio effects to optimise fusion power plant design [16], here we assess the effects of strong rotation on stability limits in tokamak plasmas.

The plasma stability is assessed using the CASTOR-FLOW code [17] using an equilibrium derived from the DIVA code [18], which treats the equilibrium toroidal rotation self-consistently. Whilst treating the rotation only in the stability analysis can be a valid approximation at very sub-Alfvénic rotation velocities, it can also lead to significantly misleading, or even erroneous results [19]. The stability of over 6000 equilibria with different pres-

ures, toroidal rotation velocities and profiles have been tested. The reference equilibrium is chosen to be an ITER-like case, with pressure and current density profiles as in the ITER baseline scenario [22] with a free-boundary solution constrained by the poloidal field currents and coil positions of ITER. The minimum safety factor is chosen to be $q_{min} \in [1.2, 1.7]$ in order to avoid introducing the $q = 1$ surface as the rotation is varied ($q_{min} > 1$ is anyway assumed for future power plants [23]). Here $q = d\psi_\phi/d\psi_\theta$ and ψ_ϕ and ψ_θ are the toroidal and poloidal magnetic fluxes. As the rotation is varied, both I/aB and the safety factor on axis are fixed since the β -limit varies with the former [20] and changes in the latter would result in a variation of the most unstable eigenmode. Since the poloidal field coil currents are also held fixed and the rotation incurs a centrifugal shift of the magnetic axis, this means that I and B are varied slightly. When the safety factor is allowed to vary it is seen that the rotation causes q_0 to drop, with the decrease dependent upon β . This can be attributed to the dynamical enhancement of β_p by the rotation [21] and a small flow-induced variation in the plasma shape.

The static β -limit for the ITER-like case with a fixed $q_0 = 1.4$ is found to be $\beta_N = 2.05$, at which point the equilibrium becomes unstable to an external kink (XK) mode. The stability of flowing equilibria is then tested for various toroidal rotation profiles, $\Omega(r)$, with different Alfvénic Mach number at the magnetic axis, $\mathcal{M}_A(0) = \Omega(0)/\omega_A(0)$, and different rotation shears, for increasing plasma pressures. Here the Alfvén frequency is $\omega_A^2 = B_0^2/\mu_0\rho_0 R_0^2$, ρ is the mass density and R_0 is the major radius of the magnetic axis. The toroidal rotation profile is taken to be of the form $\mathcal{M}_A(\psi) = \mathcal{M}_A(0)(a_0 + a_1/[1 + \exp((\psi_\theta - a_2)/a_3)])$ where a_{0-3} are real numbers. This allows a range of rotation profiles, from rigid or linearly sheared to experimental cases or ITER predictions [24]. Figure 1 shows the fractional increase in the β_N -limit compared to the static case for two different ITER equilibria with $q_0 = 1.42, 1.84$ as

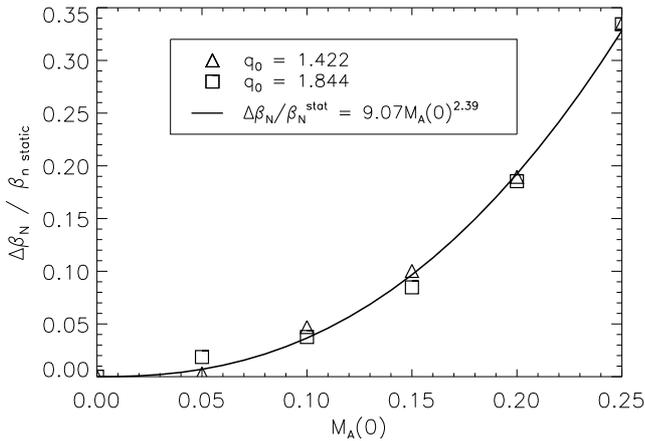


FIG. 1: The fractional increase in β -limit versus $\mathcal{M}_A(0)$ for different q_0 .

$\mathcal{M}_A(0)$ is increased for a linearly sheared profile. For rotation speeds of a quarter of the Alfvén speed, which are achievable in present-day STs, the stability limit to the XK mode has increased by a third. The exponential increase in β -limit is independent of the q -profile nuances, provided no ancillary rational surfaces are introduced, or $\Delta q = q_{min} - 1$ does not approach zero, driving infernal modes [25]. It is critical that the equilibrium flow is treated consistently, or the stabilisation of the XK mode through dynamic modification of the profiles is not fully captured. Unfortunately it is not possible to produce a β_N scaling law for arbitrary rotation profiles and amplitudes because at high rotation speeds Kelvin-Helmholtz (KH) modes [14] are driven unstable by the flow shear, providing an upper bound on stability. Furthermore, even at modest rotation speeds, the marginally stable XK mode couples to the KH mode, making the inference of the pressure limits non-trivial. However, having tested a range of rotation profiles, the linearly sheared profile always has the weakest stabilising effect on the XK mode, so the ‘minimum’ increase in the external kink mode stability limit due to sheared rotation (including the modification of the flowing equilibrium) for $R/a = 3$ plasmas is given by

$$\beta_N^{\text{low}} = \beta_N^{\text{static}}(1 + 9.07\mathcal{M}_A(0)^{2.39}) \quad (1)$$

provided that the minimum in the safety factor is above the infernal mode threshold (ie q_{min} is well above unity) and that the rotation shear is below the threshold for destabilising a KH mode, as discussed later. To use equation 1 for extrapolation to future devices, an expression for β_N^{static} can be taken from scalings with aspect ratio, I_p/aB , shaping, pressure peaking, internal inductance and so on, as given in reference [16] for instance.

KH modes have been an ephemeral concern in tokamak research in the past [26–28] though their empirical absence has allayed this (it should be noted, however, that

in NBI-heated plasmas the inherent coupling between rotation and β complicates the unambiguous determination of pressure or flow-driven instabilities). However, in future devices, such as the proposed ST-CTF, the rotation is likely to be far in excess of present tokamaks. Numerically, the KH mode is shown to be driven by the rotation shear. In a large-aspect ratio, circular plasma, $q_0 = 1.3$, $\beta_N = 1.3$ (which is below the pressure limit in the absence of a close-fitting wall) with the rotation profile approximately a Heaviside step function but with finite gradient, the $n = 1$ KH mode is driven unstable above a critical rotation threshold, in this case $v = 0.1\omega_A$, with a global eigenmode peaked at the position of the step in the rotation. Here n is the toroidal periodicity of the wave. Figure 2 shows the displacement from linear stability analysis for a stepped rotation profile when the position of the step is moved radially outwards. Whilst the position of the step influences the growth rate, it does not change the critical rotation threshold. Notably, even moving the strong flow gradient across a rational surface does not affect the critical rotation to drive the mode. However, as the rotation step crosses a rational surface (in this case at $q = 2$) the frequency of the mode changes sign, as predicted by cylindrical analysis [26]. The position of the largest gradient in the rotation thus wholly determines the mode structure, frequency and direction of propagation, but does not strongly affect the growth rate or critical rotation. The KH mode is approximately independent of pressure, and has a finite growth rate as $\beta \rightarrow 0$. Higher toroidal mode number (n) KH modes usually have larger growth rates, as illustrated in figure 3, though the eigenstructure for increasing n becomes markedly more radially localised around the step, meaning that such mid- n KH modes are unlikely to represent a significant performance limitation.

The rotation driven instability can be modelled analytically using the theory for MHD modes in toroidally rotating plasmas developed in [29]. This theory is based on the ordering $\omega \sim \Omega \sim c_s \sim \epsilon\omega_A$ and $\beta \sim \epsilon^2 \ll 1$ and assumes that there exists an extended region of low magnetic shear ($s = r/qdq/dr$) in the plasma where $m/q - n \sim \epsilon$. Here c_s is the ion sound speed, $\epsilon = a/R$ is the inverse aspect ratio and m is the poloidal mode number. The equation for a perturbation $\xi_{m,n} \sim e^{i(m\theta - n\phi - \omega t)}$ has, in the low-shear region, the form

$$(\mathcal{L}_{m,n} + \mathcal{T}_{m,n})\xi_{m,n} + r^2(m^2 - n^2)\frac{d\tilde{\beta}_0}{dr}\xi_{m,n} - m^2\frac{d\tilde{\beta}_0}{dr}C_+r^{m+1} = 0 \quad (2)$$

where $\mathcal{L}_{m,n}$ is the usual cylindrical tokamak operator and $\mathcal{T}_{m,n}$ is an operator including the effects of the plasma inertia as well as the main effects from the plasma rotation [29]. Additional effects of the rotation are, however, also present in $\tilde{\beta}_0 = 2\mu_0(p_0 + \rho_0\Omega^2R_0^2/2)/B^2$ which represents the ‘effective’ beta, including both the static pressure

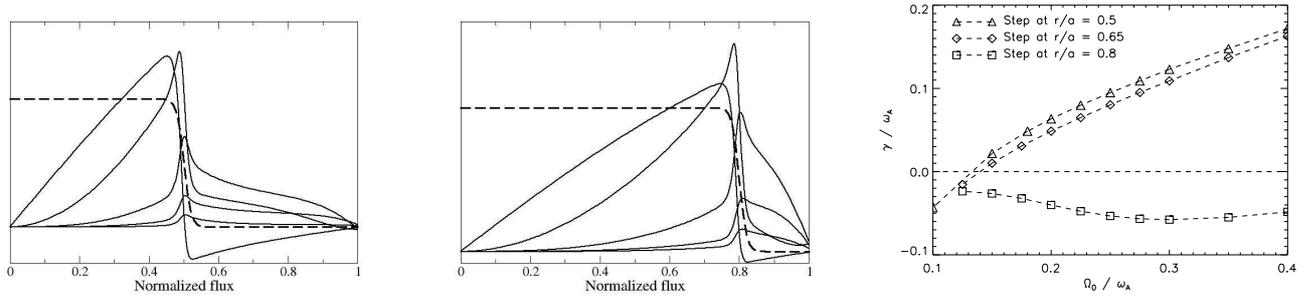


FIG. 2: The KH eigenfunctions when the flow profile step is moved radially outwards from (a) $r/a = 0.5$ to (b) $r/a = 0.8$. The flow profile is shown by the dashed lines. The $m = 1$ and $m = 2$ poloidal harmonics provide the largest components to the perturbation. Whilst the critical rotation is unchanged as the step in the flow profile moves radially outside the $q = 2$ rational surface, (c) the mode frequency changes sign.

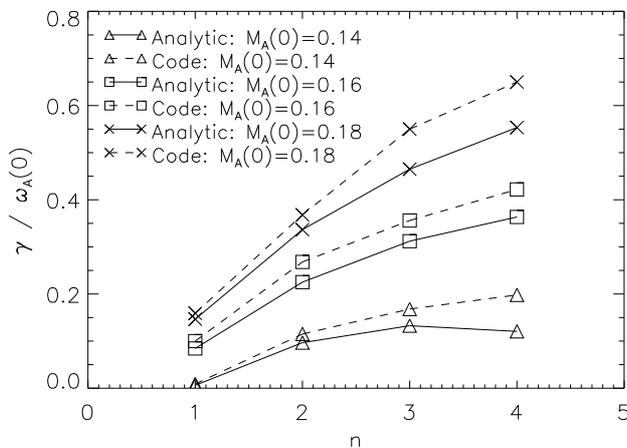


FIG. 3: KH growth-rate for different toroidal mode numbers at three $\mathcal{M}_A(0)$ as calculated using the CASTOR-FLOW code and calculated analytically for a circular equilibrium with $\epsilon = 0.1$ and $q_0 = 1.1$.

and the dynamical pressure from the rotation. The constant C_+ denotes the amplitude of the homogeneous part of the poloidal side-band $\xi_{m+1,n}$ in the low-shear region where Eq 2 is valid, and is determined by matching the solution in that region to the solution in the region of large magnetic shear:

$$C_+ = -\frac{\Lambda_{m,n} m^2 R_0^2}{n^2 r_1^{2m+2}} \int_0^{r_1} r^{m+1} \frac{d\tilde{\beta}_0}{dr} \xi_{m,n} dr \quad (3)$$

where $\Lambda_{m,n} = (m+1)(m+2+C)/(2m-2C)$, $C = r_1 \xi'_{m+1,n} / \xi_{m+1,n}$ (with $\xi'_{m+1,n}$ evaluated at $r = r_1 + 0$) and r_1 is the radius that separates the region of low shear from the region of high shear [30]. While the middle term in Eq. 2 represents the stabilizing Mercier effect (for $m > n$), the C_+ term usually has the opposite sign (for $\tilde{\beta}'_0 < 0$) and is the driving term for infernal and quasi-interchange instabilities in static, low-

shear plasmas [30]. Although a sheared rotation-driven, KH type of instability is associated already with the homogeneous part of Eq. 2, it is seen that, since $\tilde{\beta}_0$ includes also the dynamical pressure, the rotation shear contributes with an infernal mode drive through the C_+ term, an effect that increases the growth rate of the KH instability. Good agreement between analytic theory and numerical modelling is illustrated in figure 3 for a large-aspect ratio $\epsilon = 0.1$, circular equilibrium with $q_0 = 1.1$ and $\beta = 0.005$ and a rotation profile given by $\Omega(r) = \frac{1}{2}\Omega_0 + \frac{1}{2}\Omega_0 \tanh\{[(0.5a)^2 - r^2]/(0.15a)^2\}$.

It is possible to examine windows of operating space in terms of rotation amplitude and shear for a given equilibrium q_0 and β_N . By using a rotation profile with the maximum shear located at the mid-radius, and varying the parameters a_{0-3} and $\mathcal{M}_A(0)$, a range of flowing free-boundary ITER equilibria with fixed $q_0 = 1.42$ and fixed I/aB have been tested for linear stability to the XK and KH modes. Figure 4 shows the stable operating space for both $\beta_N = 2.15, 2.25$, which are respectively 5% and 10% above the ideal no-wall limit. It is evident that increasing the flow above $0.1\omega_A$ stabilises the external kink above the no-wall limit. A stronger rotation shear stabilises the XK mode more effectively than a linearly sheared profile, but at the same time is more likely to drive the KH mode unstable as the rotation amplitude increases. At $\beta_N = 2.25$ the XK mode is found to be more unstable at the highest rotation shear because of a coupling between the XK mode and the marginally-stable KH mode. A window of stability exists with optimised rotation profile and amplitude whereby operation well in excess of the ideal no-wall limit can be achieved. Access above the static pressure limits is replicated in the presence of an ideal wall as well, meaning that a combination of rotation speed and shear allows stable plasmas *above* the with-wall stability limits. This may be of considerable importance for future fusion power plants which are predicated upon achieving extremely high plasma pressures [2]. In

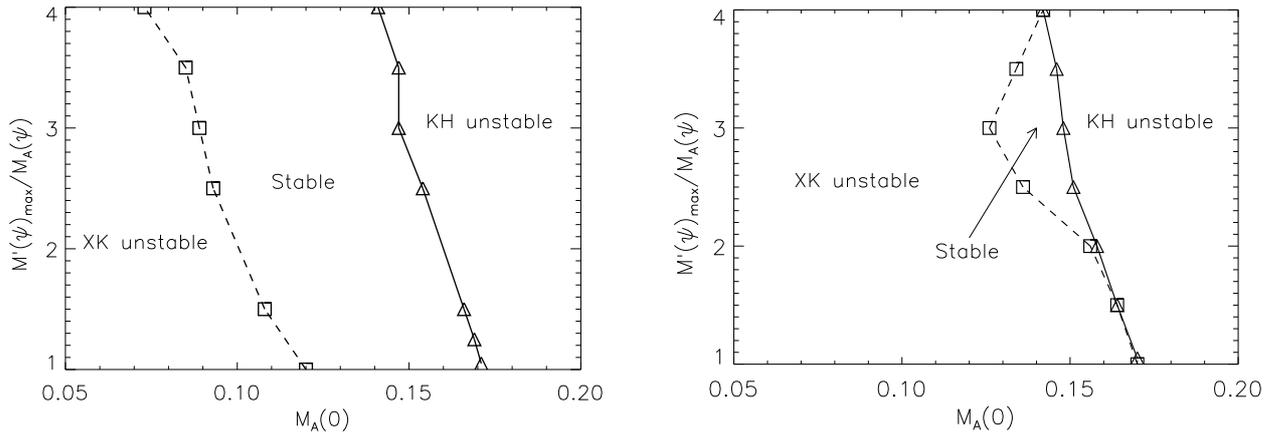


FIG. 4: The operational space of rotation shear versus amplitude for the XK and KH limits at (left) $\beta_N = 2.15$ and (right) $\beta_N = 2.25$ for an ITER equilibrium with $q_0 = 1.4$. The KH limit is almost independent of β_N .

ITER, however, the rotation speed is predicted to reach only $v_0/c_s = 0.085$ [24] meaning that the stability limits may be lower than achievable in modern-day tokamaks with stronger NBI-induced flows.

In contrast, spherical tokamaks can rotate at large fractions of the Alfvén speed. For a typical MAST plasma at low aspect ratio, the critical rotation threshold for KH onset is calculated to be approximately $0.4\omega_A$. The upper-bound to the plasma rotation or local plasma rotation shear set by the KH mode has significant implications for the operation of an ST-CTF which is likely to rotate at a large fraction of the Alfvén speed if the NBI power required to heat the plasma is unidirectional. Whilst the KH mode limits the rotation, balancing the beams to avoid KH modes may then make non-inductive current drive requirements more challenging. The KH limit is dependent upon the inverse aspect ratio, the wall position and the toroidal mode number as well as the flow amplitude and shear; conversely it is relatively independent of β and the adiabatic index. Resistivity, viscosity and kinetic effects are also likely to play a role in KH mode dynamics.

Sheared toroidal rotation is found to increase the ideal external kink stability limit, with stronger shear resulting in enhanced stabilisation. However, for toroidal rotation approaching a significant fraction of the Alfvén speed, the flow shear drives a Kelvin-Helmholtz global instability, setting an upper limit on the rotation speed for stable plasma operation. Optimising the rotation profile to maximise the pressure before encountering external kink modes, but simultaneously avoiding flow-driven instabilities, can lead to a window of stability attractive for operating future high-performance fusion devices.

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