Positron Creation and Annihilation in Tokamak Plasmas with Runaway Electrons

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It is shown that electron-positron pair production is expected to occur in post-disruption plasmas in large tokamaks, including JET and JT-60U, where up to about 10^{14} positrons may be created in collisions between multi-MeV runaway electrons and thermal particles. If the loop voltage is large enough, they are accelerated and form a beam of long-lived runaway positrons in the direction opposite to that of the electrons; if the loop voltage is smaller, the positrons have a lifetime of a few hundred ms, in which they are slowed down to energies comparable to that of the cool ($\lesssim$ 10 eV) background plasma before being annihilated.

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Electrons are often accelerated to energies of tens of MeV by the electric field induced during the disruptive instability in tokamaks [1]. The resulting beam of "runaway" electrons can carry up to about half the original plasma current. At these high energies, electron-positron pairs can be created in collisions between the runaway electrons and background plasma ions and electrons. In this Letter, we estimate the number of such pairs and discuss the fate of the positrons created in this way. Perhaps surprisingly, it appears that tokamaks may be the largest repositories of positrons made by man.

Electron runaway is possible because the friction force experienced by a fast electron falls off with increasing energy. An applied electric field may therefore lead to free-fall acceleration of energetic electrons if the electric force exceeds the limiting friction force on ultrarelativistic electrons [2],

$$E > E_c = \frac{n_e e^3 \ln\Lambda}{4\pi\epsilon_0^2 m_e c^2},$$

where n_e, e, and m_e denote the electron density, charge and rest mass, respectively, and $\ln\Lambda$ is the Coulomb logarithm. If the electric field exceeds the Dreicer field $E_D = (m_e c^2/T_e)E_c$, where T_e is the electron temperature, massive runaway of the thermal bulk population of electrons occurs. This is rarely the case in tokamaks, where the inequalities $E_c < E < E_D$ usually hold in disruptions.

Disruptions in the Joint European Torus (JET) [3] and JT-60U [4] can lead to runaway electron currents above 1 MA. Occasionally this current remains stable following the disruption and can then persist for several seconds, see Fig. 1. The energy of the runaways is in the range 10–20 MeV, while the temperature of the background plasma is less than about 10 eV. The plasma density is uncertain because of the unreliability of interferometry following a disruption; due to the influx of neutral atoms from the walls it is probably much larger than the predisruption density [5], which is of the order of 5×10^{19} m^{-3} in JET and slightly lower in JT-60U for the discharges considered here. The runaway current decays approximately exponentially in JET, with a decay time of about 2 s, if the current column is vertically stable. The electric field induced by this decay is comparable to E_c, and it is therefore possible that new runaways may be created long after the disruption. The runaway current decay rate is determined by the balance between this process and the slowing down of runaways by collisional and radiative friction [6]. In JT-60U, the runaway current can be kept constant for several seconds by increasing the loop voltage above its predisruption value, in practice above the critical field E_c. New runaways are then generated continuously by an avalanche process throughout the duration of the postdisruption discharge [4].

Since the typical runaway energy exceeds 3 times the electron rest mass, electron-positron pair production can occur in collisions between fast runaways and plasma ions [7]. Pair production also occurs in collisions between runaways and thermal electrons if the runaway energy is more than 7 times the rest mass. The expected runaway production, energy loss, and annihilation rates in the environment of large tokamaks can be estimated as follows.

At high energies, the cross section for pair production is approximately [7–9]

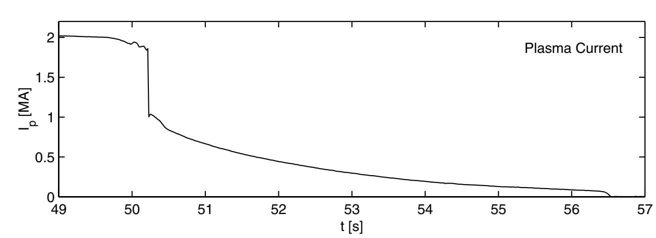

FIG. 1. Current (I_p) history in JET discharge 13340. In the disruption, which occurs at $t \simeq 50$ s, a large fraction of the plasma current is converted into a current of runaway electrons, which persists and decays gently for about 6.5 s. The runaway column remained vertically stable in this pulse, which predates the pumped divertor installation.

$$\sigma_p^s \simeq \frac{28(Z_s \alpha r_e)^2}{27\pi} \ln^3 \gamma_e, \qquad (\gamma_e \gg 1), \tag{1}$$

where $\alpha = e^2/4\pi\epsilon_0\hbar c \simeq 1/137$ is the fine-structure constant, Z_s the charge of the stationary particle (ion or electron, $s = i$ or e), $r_e = e^2/4\pi\epsilon_0 m_e c^2$ the classical electron radius, and $\gamma_e = \text{energy}/m_e c^2$ the Lorentz factor for the fast electron. Since the number of relativistic electrons in a plasma with a runaway current equal to I_r is $n_r = 2\pi R I_r/ec$, with R the major radius, the positron production rate in JET or JT-60U is approximately

$$S_p = n_r(n_e\sigma_p^e + n_i\sigma_p^i)c \simeq 4\pi R I_r \sigma_p^e n_e/e$$
$$\simeq 6.5 \times 10^{13}\ \mathrm{s}^{-1}$$

for $I_r = 1$ MA, $R = 3$ m, $n_e = 5 \times 10^{19}$ m^{-3}, and $\gamma_e \simeq 30$. This is probably an underestimate since we have only taken into account collisions with electrons and hydrogen ions. In addition, there is also an unknown but significant number of impurity ions that enter the plasma in a disruption [10]. Although these ions are in low ionization states at $T_e \lesssim 10$ eV, their full nuclear charge should be included in the pair production cross section (1), making their contribution substantial.

Most of the positrons are born with Lorentz factors in the range $1 \ll \gamma \ll \gamma_e$ since the differential cross section for pair production is proportional to $d\sigma_p/d\gamma \propto (\ln^2\gamma)/\gamma$ [9]. At birth, most of the positrons are thus strongly relativistic, and if the electric field exceeds E_c they therefore experience runaway acceleration in the direction opposite to that of the electrons. The annihilation cross section is equal to [8]

$$\sigma_a = \frac{\pi r_e^2}{1+\gamma}\left[\frac{\gamma^2+4\gamma+1}{\gamma^2-1}\ln(\gamma+\sqrt{\gamma^2-1}) - \frac{\gamma+3}{\sqrt{\gamma^2-1}}\right] \tag{2}$$

for collisions with stationary electrons. The lifetime of a high-energy positron ($\gamma \gg 1$) is thus

$$\tau_p = \frac{1}{n_e c \sigma_a} \simeq \frac{\gamma}{\pi r_e^2 n_e c \ln\gamma},$$

and exceeds the duration τ_d of a typical post-disruption discharge. The number of runaway positrons in JT-60U at the end of the postdisruption phase can therefore be estimated as

$$N_p \sim S_p \tau_d \sim 10^{14}. \tag{3}$$

To our knowledge, this number exceeds the number of positrons in any other man-made device except a nuclear explosion.

On the other hand, if $E < E_c$ after the disruption, the positrons do not run away but lose most of their energy fairly quickly—in general before they are annihilated in collisions with electrons. There are two principal energy loss mechanisms: radiation reaction and collisional friction. Because of gyromotion in the magnetic field, positrons emit synchrotron radiation and are slowed down at the rate [6]

$$\dot{p}_{\rm rad} = -\frac{p_\perp^2\sqrt{1+p^{-2}}}{\tau_r},$$

where $p = \gamma v/c = (\gamma^2-1)^{1/2}$ is the relativistic momentum normalized to $m_e c$, $p_\perp$ is its component perpendicular to the magnetic field, and $\tau_r = 6\pi\epsilon_0 m_e^3 c^3/e^4 B^2$ with B the magnetic field strength. In addition, collisional friction with the plasma electrons slows down a positron at the rate

$$\dot{p}_{\rm coll} = -\frac{1+p^{-2}}{\tau},$$

where $\tau = 1/4\pi r_e^2 n_e c \ln\Lambda$ is the collision time for relativistic positrons and electrons. For typical JET or JT-60U parameters, radiation emission is the dominant energy loss mechanism for $p_\perp \gtrsim 3$ since

$$\frac{\dot{p}_{\rm rad}}{\dot{p}_{\rm coll}} = \frac{2\epsilon_0 B^2}{3 n_e m_e \ln\Lambda}\frac{p_\perp^2}{\sqrt{1+p^{-2}}} \simeq \frac{0.1 p_\perp^2}{\sqrt{1+p^{-2}}}$$

if $B = 3$ T and $n_e = 5 \times 10^{19}$ m^{-3}. Thus, initially a typical positron slows down mostly because of synchrotron radiation emission; when it has reached mildly relativistic energy, collisions take over and thermalize it.

If the magnetic field is weaker or the density is higher, synchrotron radiation is less important. In this case, the positron distribution function can be calculated from the kinetic equation [2,11]

$$\frac{\partial f}{\partial t} = \frac{1}{\tau p^2}\frac{\partial}{\partial p}[(1+p^2)f] - n_e v \sigma_a f + s_p(p), \tag{4}$$

where the first term on the right describes slowing down and the second term describes annihilation. If the energy of the positron is low enough, the annihilation cross section (2) must be corrected to account for the focusing effect of Coulomb attraction between two particles, and then becomes [12]

$$\sigma_a \to \frac{\sigma_a x}{1-e^{-x}},$$

where $x = 2\pi\alpha/p$. This correction is important if $x \gtrsim 1$, i.e., if $E = m_e v^2/2 \lesssim 500$ eV. To determine the fate of a positron born with some arbitrary momentum, p_*, we construct the steady-state solution of Eq. (4) with a delta-function source, $s_p = s\delta(p - p_*)/p^2$ on the right,

$$f(p) = \frac{s}{1+p^2}\exp\left[\frac{1}{4\ln\Lambda}\int_p^{p_*}\frac{p'^3 F(p')}{(1+p'^2)^{3/2}}dp'\right] \times H(p_* - p),$$

with $F(p) = \sigma_a(p)/\pi r_e^2$ and H the Heaviside function. The exponential factor is equal to the fraction of positrons that survive the slowing down from p_* to p without annihilation. The fraction annihilated before thermalization is thus

$$1 - \exp\left[\frac{1}{4\ln\Lambda}\int_0^{p_*}\frac{p^3F(p)}{(1+p^2)^{3/2}}dp\right] \simeq \begin{cases} \pi\alpha p_*^2/4\ln\Lambda & (p_* \ll 2\pi\alpha), \\ p_*^3/12\ln\Lambda & (2\pi\alpha \ll p_* \ll 1), \\ 1-\exp(-\frac{\ln^2 p_*}{8\ln\Lambda}) & (p_* \gg 1), \end{cases}$$

and is fairly small for realistic energies; see Fig. 2. In other words, the majority of positrons slow down to thermal energy before getting annihilated, even if slowing down by the emission of radiation is neglected. At thermal energies, $p \ll 2\pi\alpha$, the annihilation cross section becomes $\sigma_a = 2\pi^2\alpha r_e^2 c^2/v^2$, so that the mean lifetime for thermalized positrons is

$$\tau_p \simeq \frac{1}{n_e v_{Te}\sigma_a} = \frac{v_{Te}}{2\pi^2\alpha r_e^2 n_e c^2} \simeq \frac{1}{4}\ s$$

if $n_e = 5 \times 10^{19}$ m^{-3}. This is comparable to the slowing down time of relativistic positrons and shorter than the decay time of the runaway electron current. The number of thermalized positrons in JET or JT-60U in the case $E < E_c$ can thus be estimated as

$$N_p \sim S_p\tau_p \sim 10^{13}.$$

Although smaller than (3), this number of positrons is still larger than that in nuclear reactors or other physics experiments (known to us), including particle accelerators and experiments using large lasers to create electron-positron plasmas [13,14].

Since neutral hydrogen atoms are likely to be present in the cool post-disruption plasma, some positrons will form positronium (Ps) through the reaction

$$e^+ + \mathrm{D} \rightarrow \mathrm{Ps} + \mathrm{D}^+. \qquad (5)$$

Since positronium has a very short lifetime ($< 1\,\mu$s), this results in rapid annihilation of positrons if the neutral atom density is high enough [15]. The cross section for the reaction (5) peaks at 10–20 eV, where it reaches $\sigma_x = 4 \times 10^{-20}$ m^2. This implies that the lifetime of a positron in this energy range is

$$\tau_p \sim \frac{1}{n_n\sigma_x v_{Te}} \simeq \frac{1.3\times 10^{13}}{n_n}\,\mathrm{m^{-3}\cdot s}.$$

The neutral density n_n in the plasma is not well known, but an upper bound can be obtained from measurements of the neutral gas pressure in the pumping ducts of the JET vacuum vessel. After the disruption, this pressure falls to about 2×10^{-4} Pa in the discharge shown in Fig. 1. If the gas pressure is constant outside the plasma, this means that the density of recycling neutral atoms at the plasma edge should be less than 3×10^{15} m^{-3} (assuming the edge neutral temperature is at least 1 eV). The neutral density inside the plasma should be even smaller, implying that the lifetime of a thermalized positron exceeds 3 ms. However, the cross section σ_x falls off rapidly with increasing energy ($\sigma_x = 3 \times 10^{-22}$ m^2 at 10 keV [15]), so that unthermalized positrons (having energies comparable to their birth energy, which is relativistic) are not likely to form Ps.

Detecting the positrons in a tokamak is complicated by the fact that bremsstrahlung from the runaway population tends to overwhelm the radiation emitted at annihilation events. The cross section for emitting a bremsstrahlung photon is larger than the pair production cross section by a factor of about $\alpha^{-1} = 137$, so that many gamma-ray photons are emitted for each positron created. On the other hand, the annihilation radiation is peaked around 511 keV and emitted fairly isotropically, while the bremsstrahlung spectrum is flat in energy but centered in a narrow cone around the velocity vectors of the runaway electrons. In addition, there are two situations in which positrons could perhaps be detected more easily. First, as already mentioned, if the toroidal electric field exceeds the critical field for runaway, E_c, some time after the disruption, as is certainly the case in JT-60U and perhaps also in JET, a population of runaway positrons would be formed. Although these positrons are relatively few, their bremsstrahlung would be peaked in the direction opposite from that of the electrons. Second, the low-temperature background plasma may persist for some short time after the runaway electron beam has collapsed. As we have seen, this plasma contains a population of relatively long-lived positrons, and it could be possible to detect their annihilation.

In summary, a substantial number of positrons can be created in post-disruption tokamak plasmas with runaway electrons. At birth, these positrons have highly relativistic energies and either experience runaway acceleration or are thermalized under the action of radiation emission and collisional friction before being

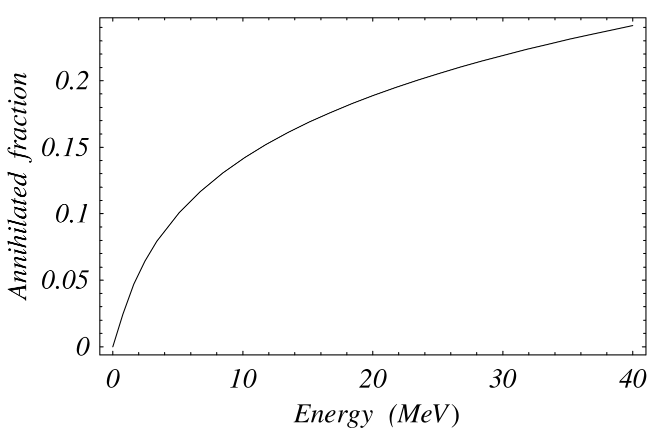

FIG. 2. Fraction of positrons that are annihilated before thermalization as a function of energy at birth.

annihilated. Runaway positrons have a lifetime of several seconds, while slowing down ones survive for a few hundred ms.

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